Millimeter Wave Link Configuration Using Out-Of-Band Information

Anum Ali

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Committee Members

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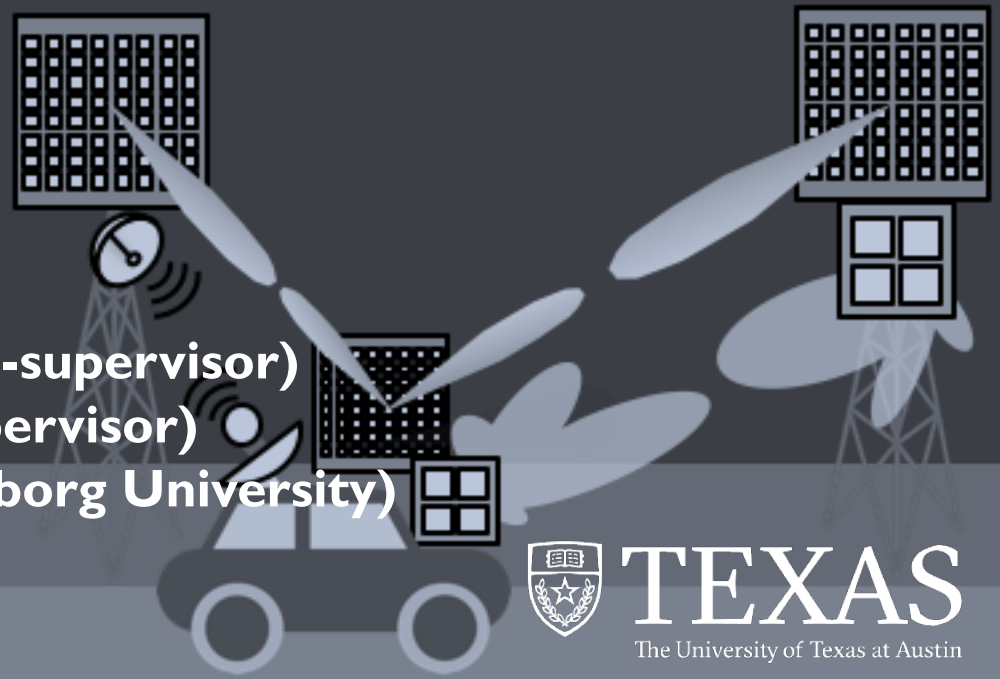
Prof. Haris Vikalo



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Some figures and icons used in this slide deck were created by Prof. Gonzalez-Prelcic.

Outline

Motivation and background

MmWave + out-of-band

1

MmWave beam-selection using sub-6 GHz information

2

MmWave covariance estimation using sub-6 GHz information

MmWave link configuration using passive radar

3

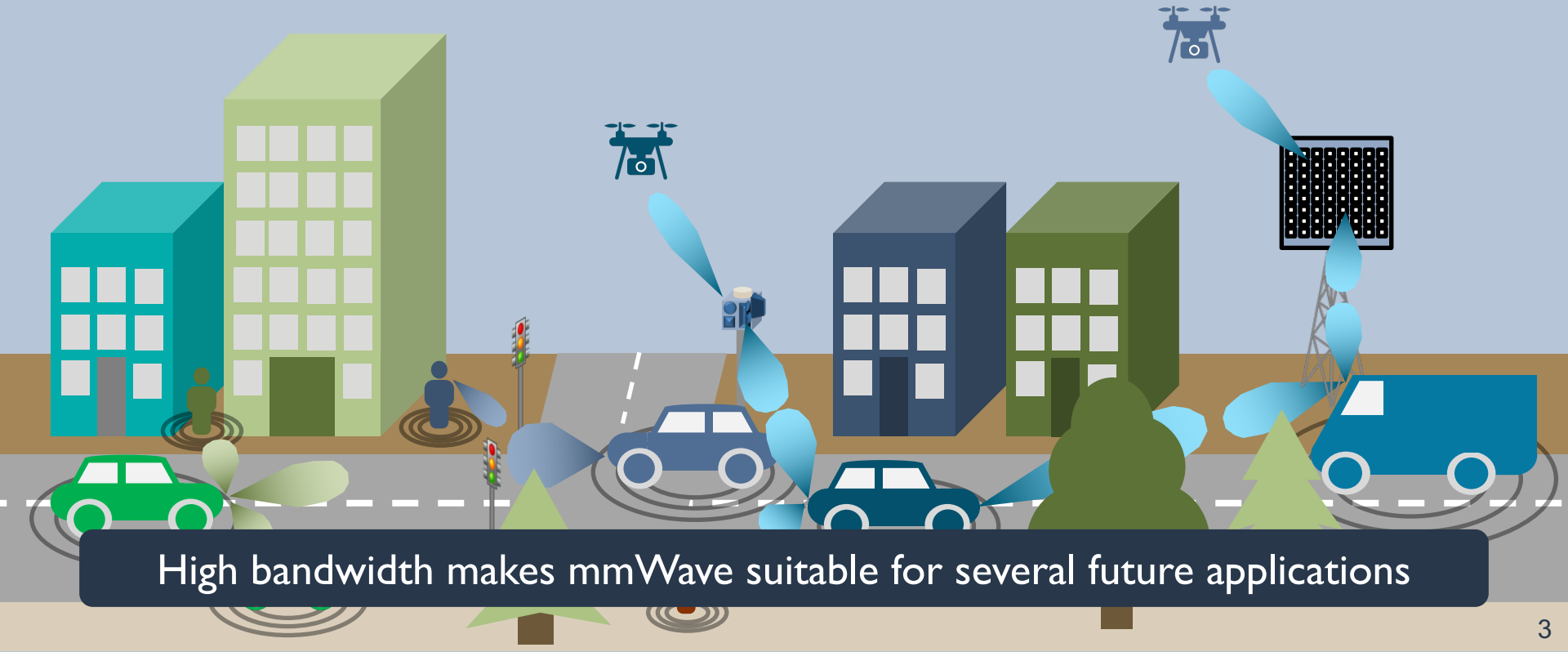
Why millimeter wave communication?

High definition 3D map download (~Gbyte)

Infotainment & cloud services

Sharing local sensor data for cooperative perception

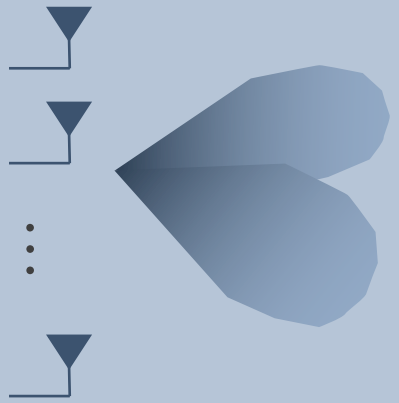
High speed internet access



High bandwidth makes mmWave suitable for several future applications

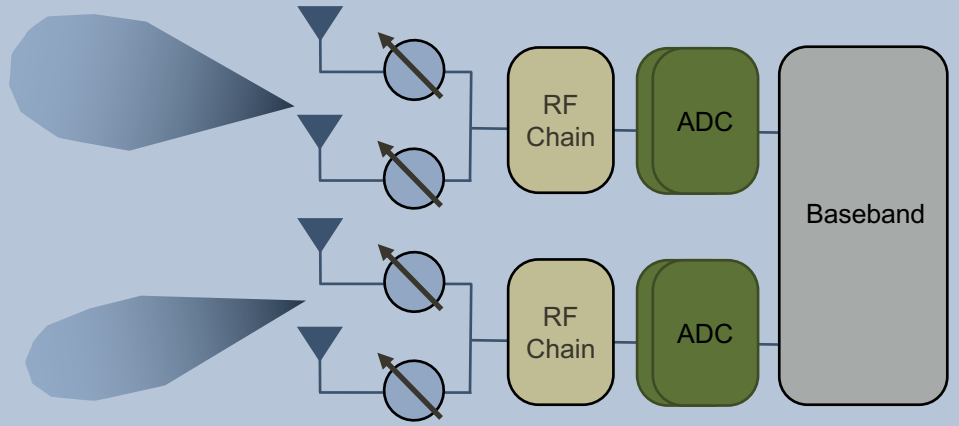
More antennas and hardware constraints

Many antennas at the TX and RX



Directional transmission/reception for sufficient link margin

Hardware constraints impact how the antennas are used

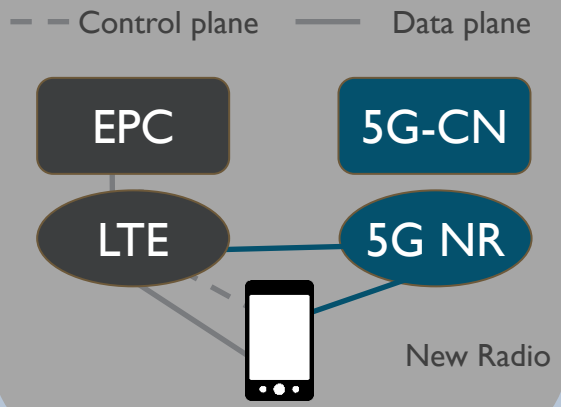


Significant fraction of channel coherence time spent in training

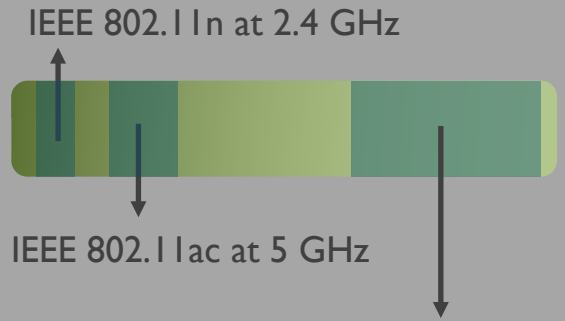
Low overhead link configuration challenging for mmWave

Out-of-band information

Cellular commun. [1]



WiFi [2][3]



V2X commun. [4]

Sub-6 GHz: difference in array size

Sensors: physical separation

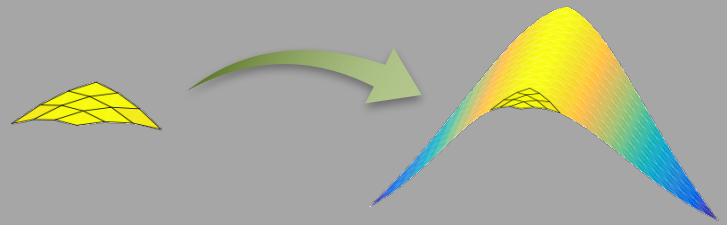
Out-of-band info. available but challenges in using it

[1] www.3gpp.org/news-events/3gpp-news/1929-nsa_nr_5g
[2] L. Verma, M. Fakharzadeh, and S. Choi, "Wifi on steroids: 802.11 ac and 802.11 ad", IEEE Wireless Commun., vol. 20, no. 6, pp. 30–35, Dec. 2013.
[3] T. Nitsche, A. B. Flores, E. W. Knightly, and J. Widmer, "Steering with eyes closed: mm-wave beam steering without in-band measurement," in Proc. INFOCOM, Apr. 2015, pp. 2416–2424.
[4] J. Choi, V. Va, N. Gonzalez-Prelcic, R. Daniels, C. R. Bhat, and R. W. Heath Jr., "Millimeter wave vehicular communication to support massive automotive sensing," IEEE Commun. Mag., vol. 54, no. 12, pp. 160–167, Dec. 2016.

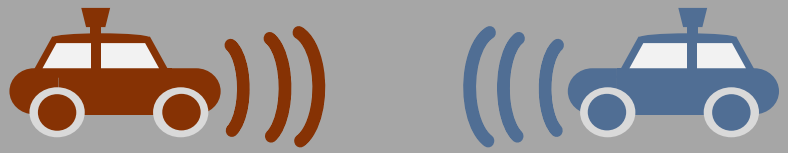
MmWave link configuration using out-of-band information

Contribution I
Beam-selection using
sub-6 GHz information

Contribution II
Covariance estimation using
sub-6 GHz information



Contribution III
Link configuration
using passive radar



Thesis statement: Out-of-band aided mmWave link configuration has a low training overhead in comparison with in-band only link configuration.

Contribution I

Millimeter wave beam-selection using sub-6 GHz information

A. Ali and R. W. Heath Jr., "Compressed beam-selection in millimeter wave systems with out-of-band partial support information," in Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP), Mar. 2017, pp. 3499–3503.

A. Ali, N. Gonzalez-Prelcic, and R. W. Heath Jr., "Millimeter wave beam-selection using out-of-band spatial information," IEEE Trans. Wireless Commun., vol. 17, no. 2, pp. 1038–1052, 2018.

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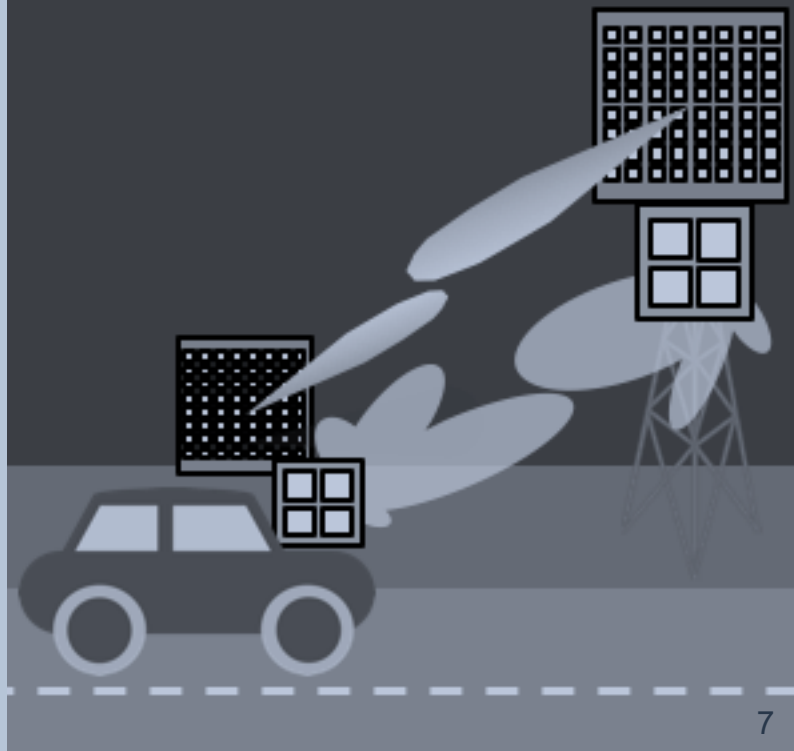
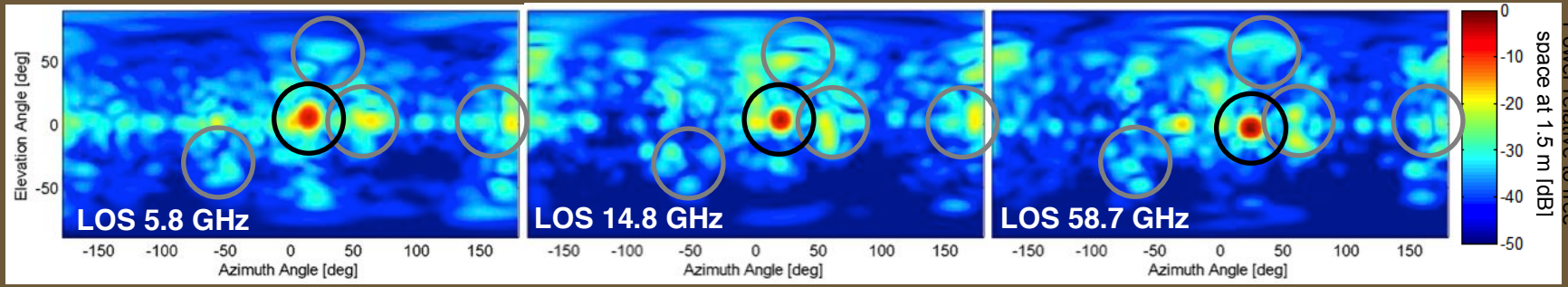


Is it possible to use sub-6 GHz info. for mmWave?

Spatial characteristics across frequencies on the order of few MHz are similar [1]

Spatial channel characteristics similar across several GHz [2]



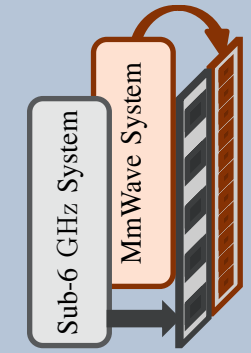
Late arriving multi-paths have more frequency dependence [3]

Number of paths, cluster power decay similar at 28 GHz and 73 GHz [4]

Spatial congruence observed in channels separated by several GHz

[1] T. Asté, P. Forster, L. Fety, and S. Mayrargue, "Downlink beamforming avoiding DOA estimation for cellular mobile communications," in Proc. ICASSP, 1998.
[2] M. Peter et al., "Measurement campaigns and initial channel models for preferred suitable frequency ranges," Millimetre-Wave Based Mobile Radio Access Network for 5 Integrated Communications, Tech. Rep., 2016.
[3] R. C. Qiu and I.-T. Lu, "Multipath resolving with frequency dependence for wide-band wireless channel modeling," IEEE Trans. Veh. Technol., vol. 48, no. 1, pp. 273–285, Jan. 1999.
[4] M. K. Samimi and T. S. Rappaport, "3-D millimeter-wave statistical channel model for 5G wireless system design," IEEE Trans. Microw. Theory Techn., vol. 64, no. 7, pp. 2207–2225, Jul. 2016.

Multi-frequency communication set-up

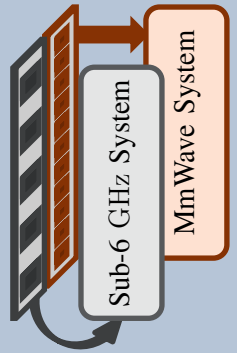


Transmitter

Multi-band MIMO system

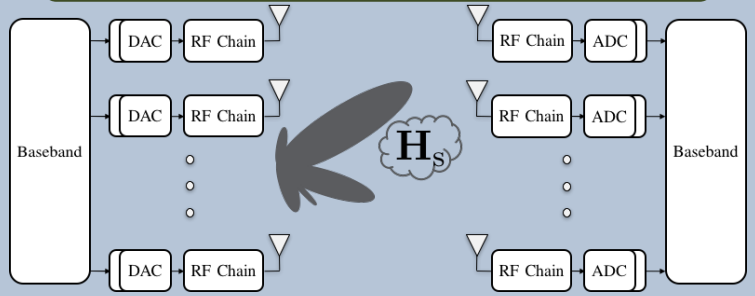
Sub-6 GHz and mmWave system operate simultaneously

Colocated, aligned and comparable aperture arrays



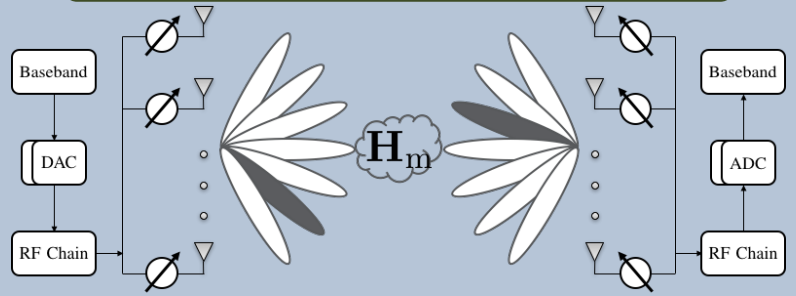
Receiver

Fully digital sub-6 GHz system



Transmitter Receiver

Analog mmWave system



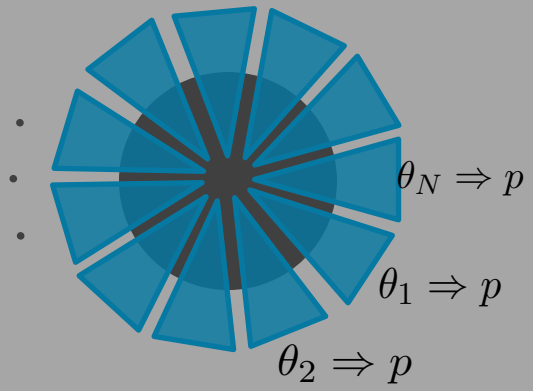
Transmitter Receiver

Key assumption: sub-6 GHz and mmWave channels are spatially congruent

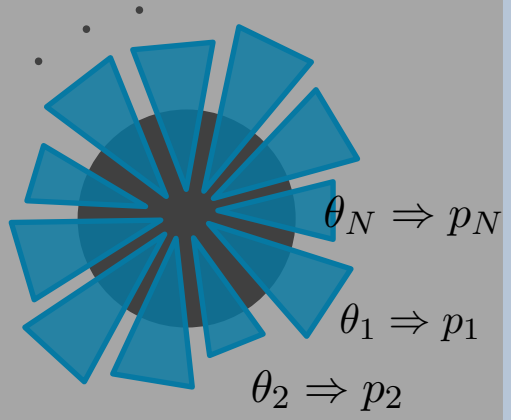
Sub-6 GHz info. in compressed beam-selection

Weighted compressed sensing

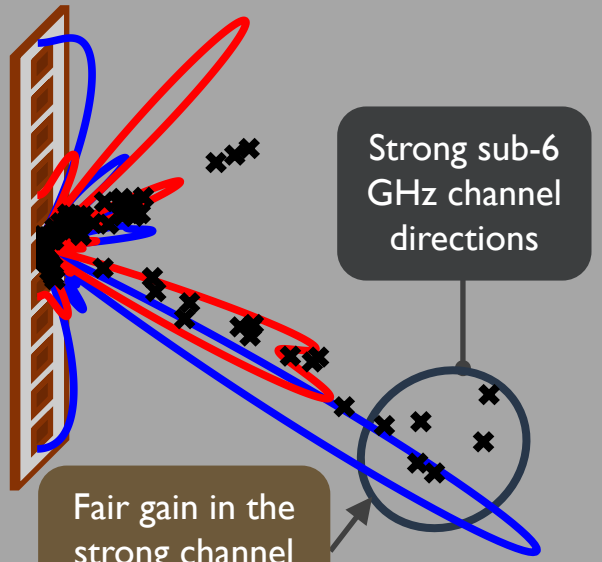
No prior information



Prior info. from sub-6 GHz

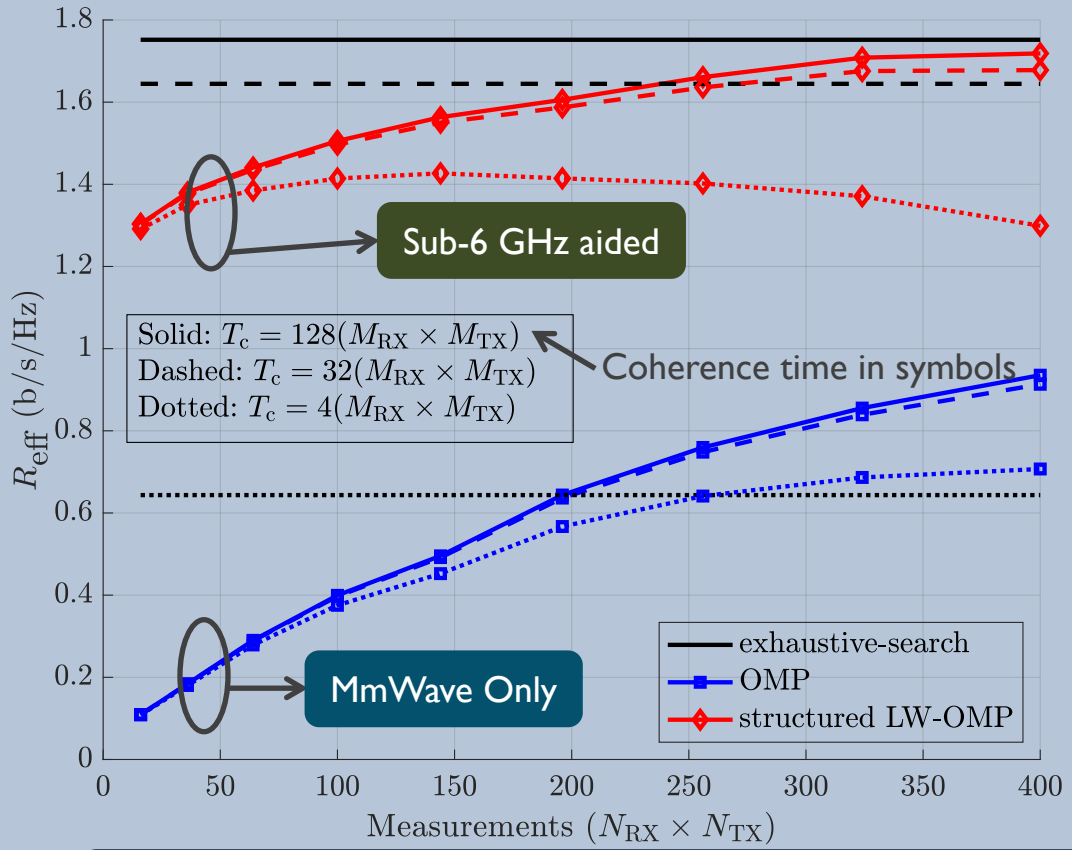


Structured random beams



Sub-6 GHz info. incorporated using weighting and structured random beams

Simulation results



Effective rate

$$R_{\text{eff}} = \left(1 - \frac{N_{\text{TX}} N_{\text{RX}}}{T_{\text{c}}}\right) R$$

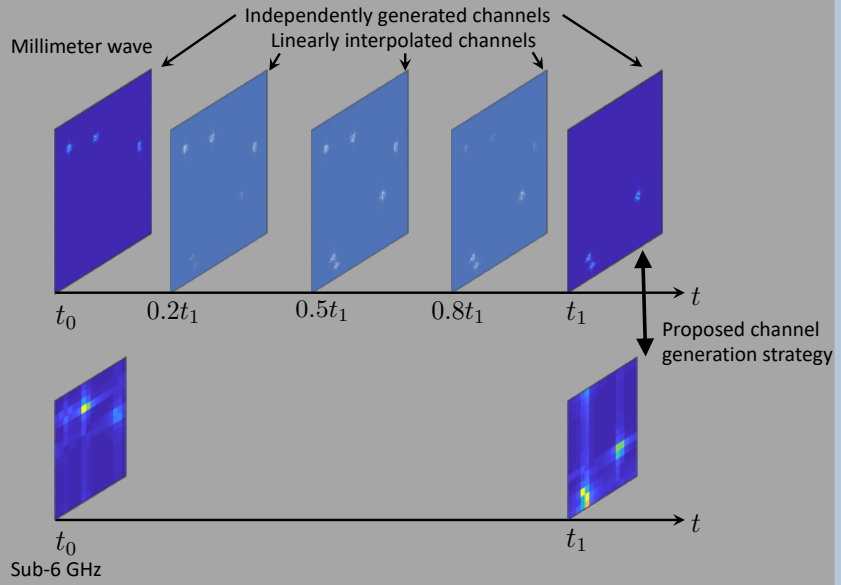
Parameter	Value
Sub-6 GHz frequency	3.5 GHz
Sub-6 GHz bandwidth	150 MHz
MmWave frequency	28 GHz
MmWave bandwidth	800 MHz
Sub-6 GHz (TX/RX) antennas	2/8
MmWave (TX/RX) antennas	16/64

Based on proposed multi-frequency channel generation method

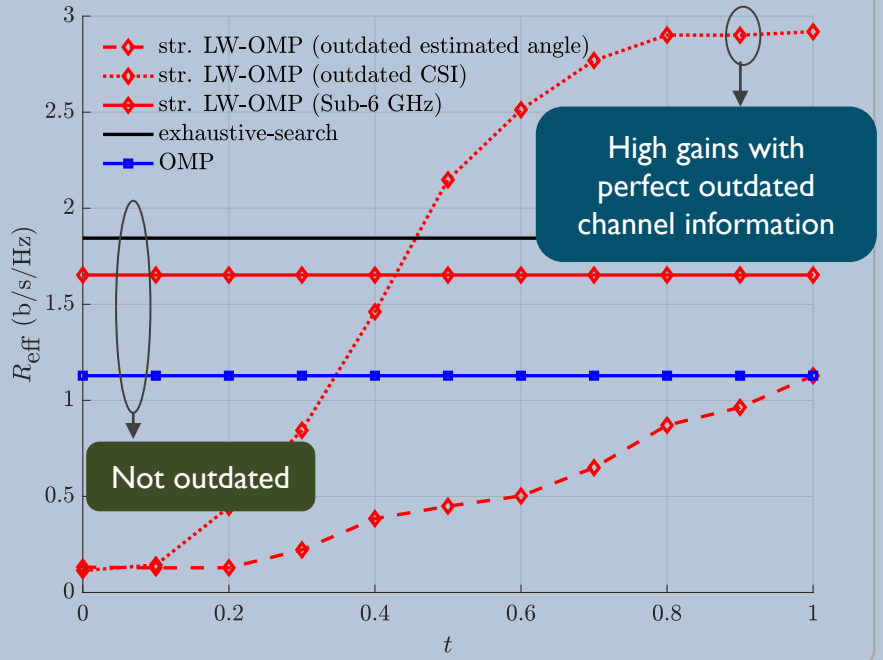
Sub-6 GHz info. reduces the overhead of exhaustive search by up to 4x

Qualifying exam feedback: Outdated information

Time evolution of channels



Result



High quality outdated information helps, but difficult to obtain

Summary

Contributions

- Proposed a multi-frequency channel model
- Formulated the compressed beam-selection problem
- Used sub-6 GHz information in beam-selection problem

Results

- Weighting and structured codebooks reduce overhead
- Up to 4x reduction compared to exhaustive search

Contribution II

Millimeter wave channel covariance estimation using sub-6 GHz information

A. Ali, N. Gonzalez-Prelcic, and R. W. Heath Jr., "Estimating millimeter wave channels using out-of-band measurements," in Proc. Inf. Theory Appl. (ITA) Wksp, Feb. 2016, pp. 1–5.

A. Ali, N. Gonzalez-Prelcic, and R. W. Heath Jr, "Spatial Covariance Estimation for Millimeter Wave Hybrid Systems using Out-of-Band Information," IEEE Trans. Wireless Commun (early access).

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MmWave channel covariance

Why estimate covariance?

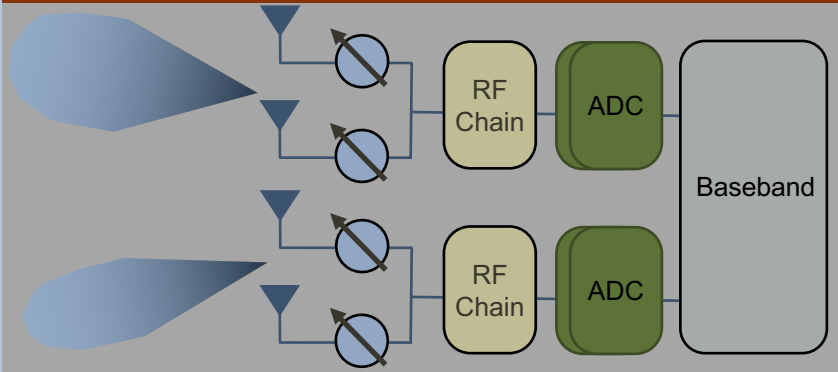
Covariance varies much slowly compared to instantaneous channel

↓

Less frequent training/low training overhead

Performance comparable to instantaneous channel knowledge in limited scattering channels [1]

Challenges in estimating covariance



Not direct access to the channel at mmWave due to hardware constraints

Sub-6 GHz covariance estimation easy

Use sub-6 GHz channel covariance to estimate mmWave channel covariance

[1] A. Alkhateeb et al., "Hybrid precoding for millimeter wave cellular systems with partial channel knowledge," in Proc. Inf. Theory Appl. (ITA) Wksp, Feb. 2013, pp. 1–5.

Sub-6 GHz to mmWave covariance translation

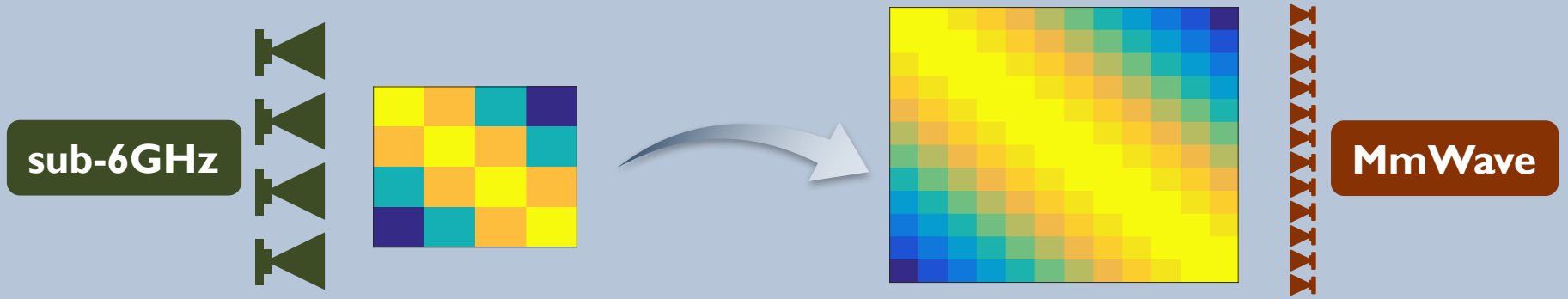
Given sub-6 GHz covariance estimate

$$\mathbf{R}_{\mathrm{S}}$$

Estimate of the mmWave cov. matrix

$$\hat{\mathbf{R}}_{\mathrm{m}} = f(\mathbf{R}_{\mathrm{s}})$$

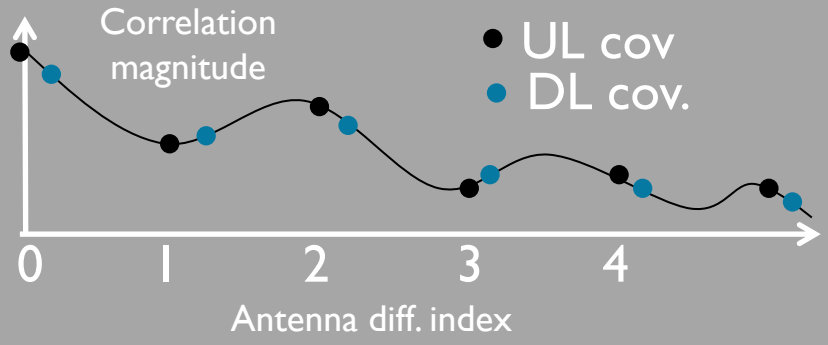
The size of the covariance matrices is very different



Dimension mismatch creates challenges in covariance translation

Related work

Interpolation-Extrapolation based [1]



Array manifold based transformation [2]

$$\mathbf{T}^{*}\mathbf{R}_{\mathrm{UL}}\mathbf{T} = \mathbf{R}_{\mathrm{DL}}$$

Transformation matrix

UL Cov. Matrix

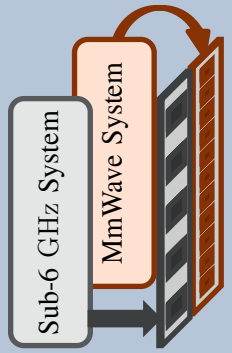
DL Cov. Matrix

Spatio-temporal covariance translation based on 2D interpolation [3]

Existing work only for arrays of similar size and small frequency separation

[1] M. B. Khalilsarai et al. "FDD Massive MIMO via UL/DL Channel Covariance Extrapolation and Active Channel Sparsification", IEEE Trans. Wireless Commun., 2018.
[2] T. Asteˊ et al., "Downlink beamforming avoiding DOA estimation for cellular mobile communications," in Proc. IEEE ICASSP, May 1998, pp. 3313–3316.
[3] M. Jordan, X. Gong, and G. Ascheid, "Conversion of the spatio-temporal correlation from uplink to downlink in FDD systems," in Proc. IEEE WCNC, Apr. 2009, pp. 1–6.

Multi-frequency communication set-up



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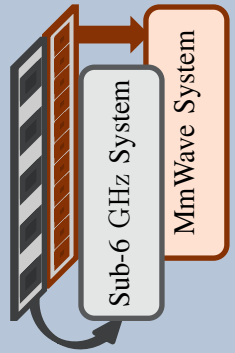
Multi-band MIMO system

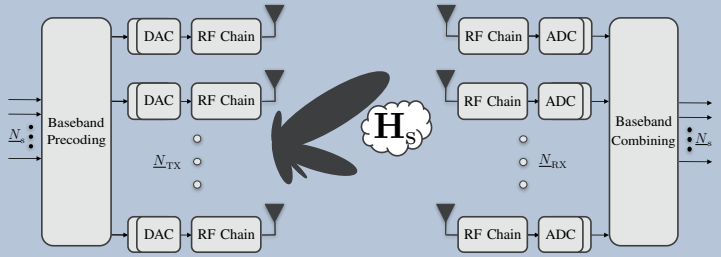
Sub-6 GHz and mmWave system operate simultaneously

Colocated, aligned and comparable aperture arrays



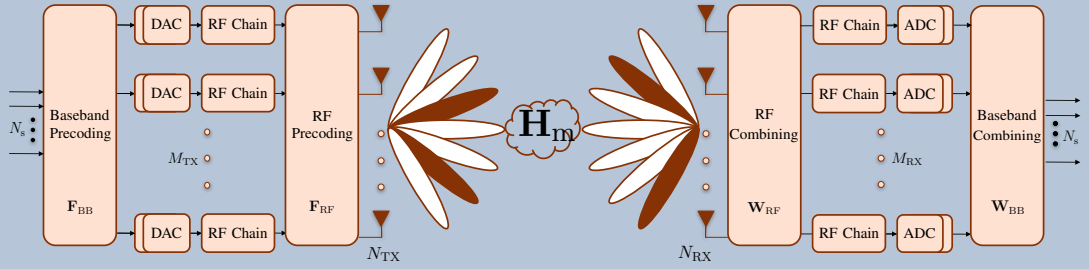
Receiver

Fully digital sub-6 GHz system



Transmitter Receiver

Hybrid mmWave system



Transmitter Receiver

Estimate mmWave covariance given sub-6 GHz covariance

Approach 1: Sub-6 GHz covariance translation

Parametric approach: estimate covariance parameters

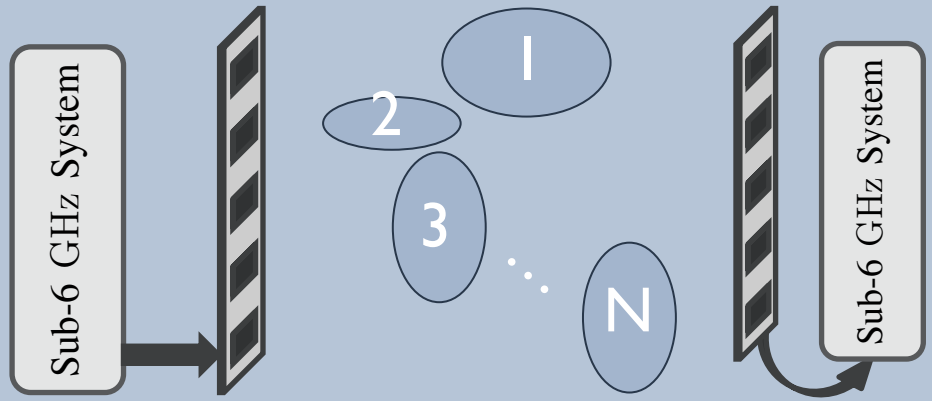
Estimate the number of clusters [1]

Minimum description length criteria

Estimate the cluster AoAs and ASs [2]

Estimate the power contribution of clusters

Construct the mmWave covariance



Only a few clusters can be estimated

[1] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," IEEE Trans. Acoust., Speech, Signal Process., vol. 33, no. 2, pp. 387–392, Apr. 1985.
[2] M. Bengtsson and B. Ottersten, "Low-complexity estimators for distributed sources," IEEE Trans. Signal Process., vol. 48, no. 8, pp. 2185–2194, Aug. 2000.

Approach 1: Sub-6 GHz covariance translation

Parametric approach: estimate covariance parameters

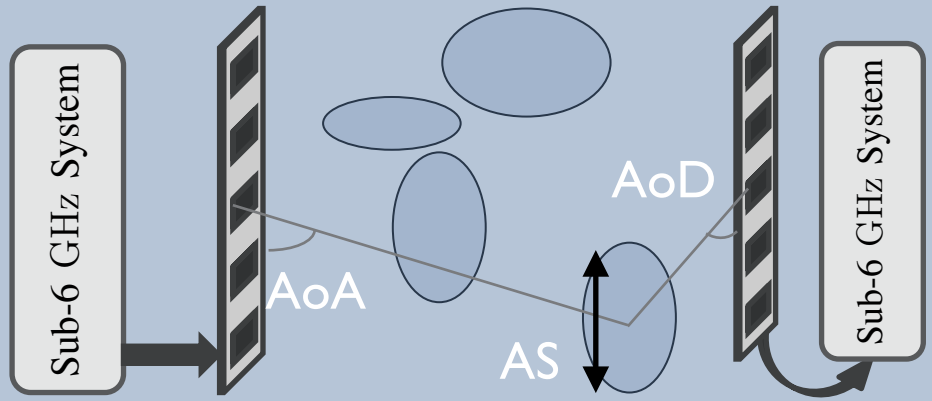
Estimate the number of clusters [1]

Spread root-MUSIC algorithm

Estimate the cluster AoAs and ASs [2]

Estimate the power contribution of clusters

Construct the mmWave covariance



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Approach 1: Sub-6 GHz covariance translation

Parametric approach: estimate covariance parameters

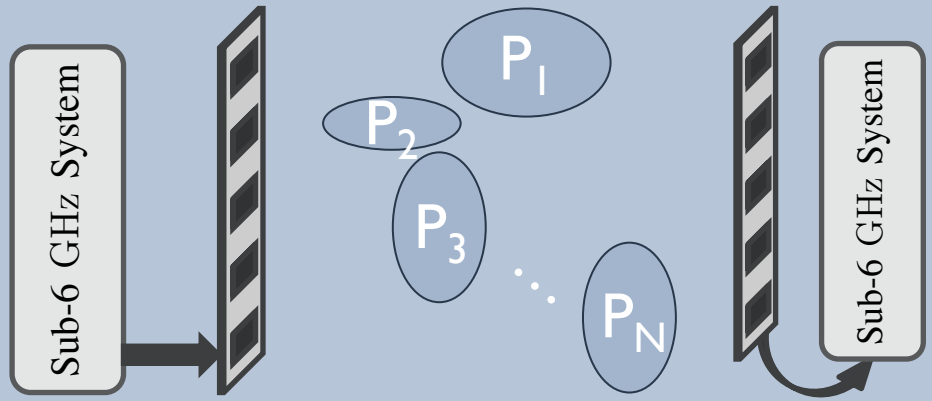
Estimate the number of clusters [1]

Estimate the cluster AoAs and ASs [2]

Estimate the power contribution of clusters

Construct the mmWave covariance

Non-negative least squares



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Approach 1: Sub-6 GHz covariance translation

Parametric approach: estimate covariance parameters

Estimate the number of clusters [1]

Estimate the cluster AoAs and ASs [2]

Estimate the power contribution of clusters

Construct the mmWave covariance

Cluster power

$$\hat{\mathbf{R}}_{\mathrm{m}} = P_1\mathbf{R}(\theta_1, \sigma_{\theta_1}) + \ldots + P_N\mathbf{R}(\theta_N, \sigma_{\theta_N})$$

Closed form expression of covariance evaluated for a the estimated AoA and ASs

Closed form expression for Gaussian PAS

$$\mathbf{R}_{i,j}(\theta, \sigma_\theta) = e^{-((i-j)2\pi\Delta\cos(\theta)\sigma_\theta)^2} e^{\mathrm{j}2\pi\Delta(i-j)\sin(\theta)}$$

Only a few clusters can be estimated

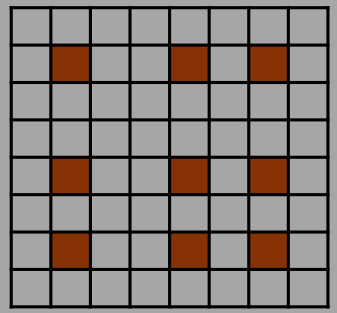
[1] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," IEEE Trans. Acoust., Speech, Signal Process., vol. 33, no. 2, pp. 387–392, Apr. 1985.
[2] M. Bengtsson and B. Ottersten, "Low-complexity estimators for distributed sources," IEEE Trans. Signal Process., vol. 48, no. 8, pp. 2185–2194, Aug. 2000.

Approach 2: Sub-6 GHz aided mmWave cov. estimation

In-band covariance estimation [1]

Limited scattering

Hermitian symmetry

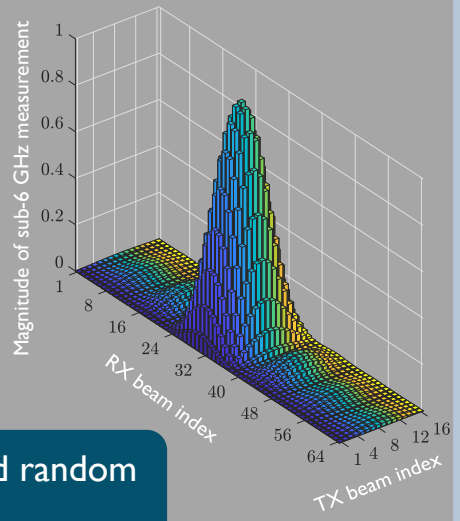


Low computational complexity alternative to [1]

Sub-6 GHz aided covariance estimation

Weighted compressed covariance estimation

Weights obtained from sub-6 GHz

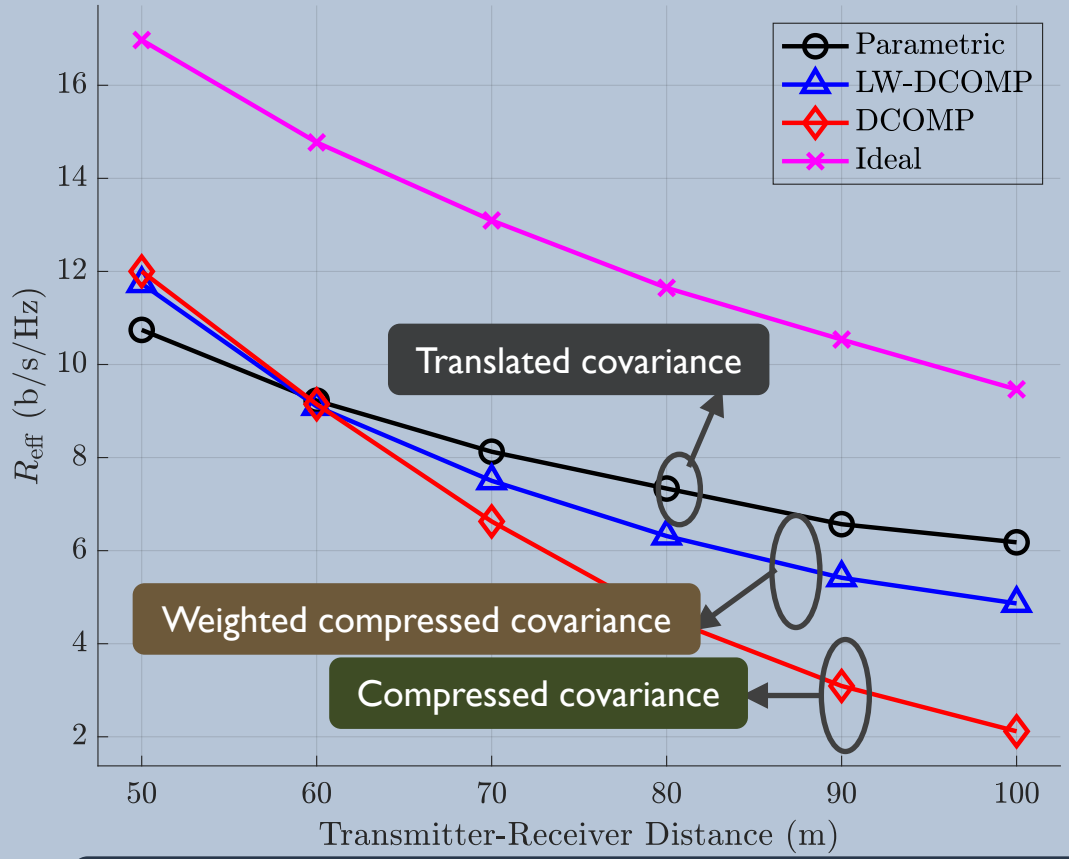


Training using structured random codebooks

Weighted compressed covariance estimation for MIMO systems

[1] S. Park and R. W. Heath Jr, "Spatial Channel Covariance Estimation for the Hybrid MIMO Architecture: A Compressive Sensing Based Approach," IEEE Trans. Wireless Commun., vol. 17, no. 12, pp. 8047– 8062, 2018.

Simulation results



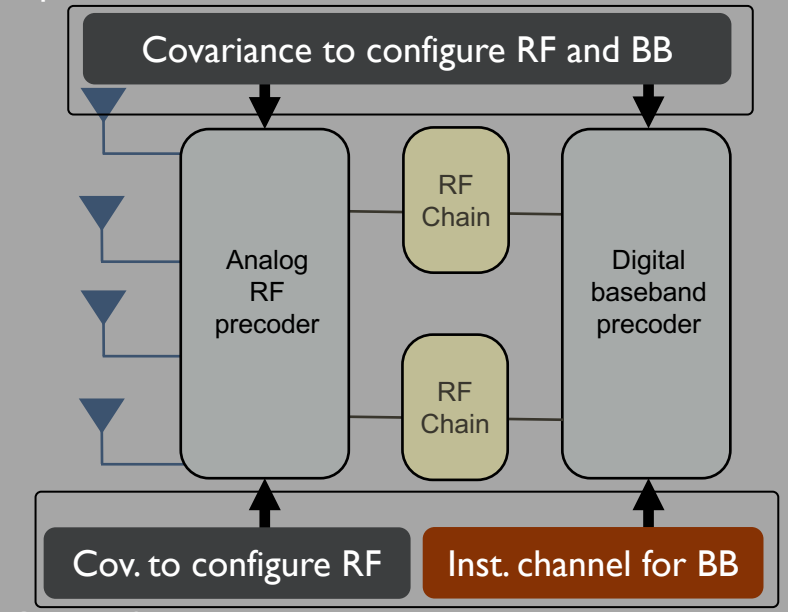
Parameter	Value
Sub-6 GHz frequency	3.5 GHz
Sub-6 GHz bandwidth	150 MHz
MmWave frequency	28 GHz
MmWave bandwidth	800 MHz
Sub-6 GHz (TX/RX) antennas	4/8
MmWave (TX/RX) antennas	32/64
MmWave (TX/RX) RF chains	8/16

Sub-6 GHz info. improves the rate, especially for large TX-RX distance

Qualifying exam feedback: How to use covariance?

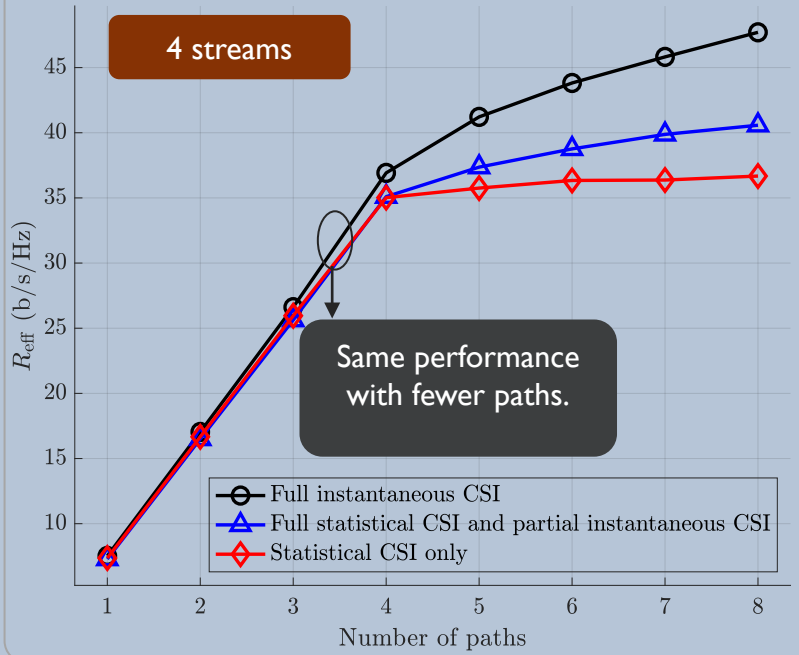
Different ways to use covariance info.

Option 1



Option 2

Result



Performance gap depends on the number of paths

Summary

Contributions

- Proposed a parametric covariance translation method
- Proposed a weighted compressed covariance estimation method

Results

- Sub-6 GHz aided covariance estimation has lower overhead
- Depending on the TX/RX distance, one strategy may be preferred over the other

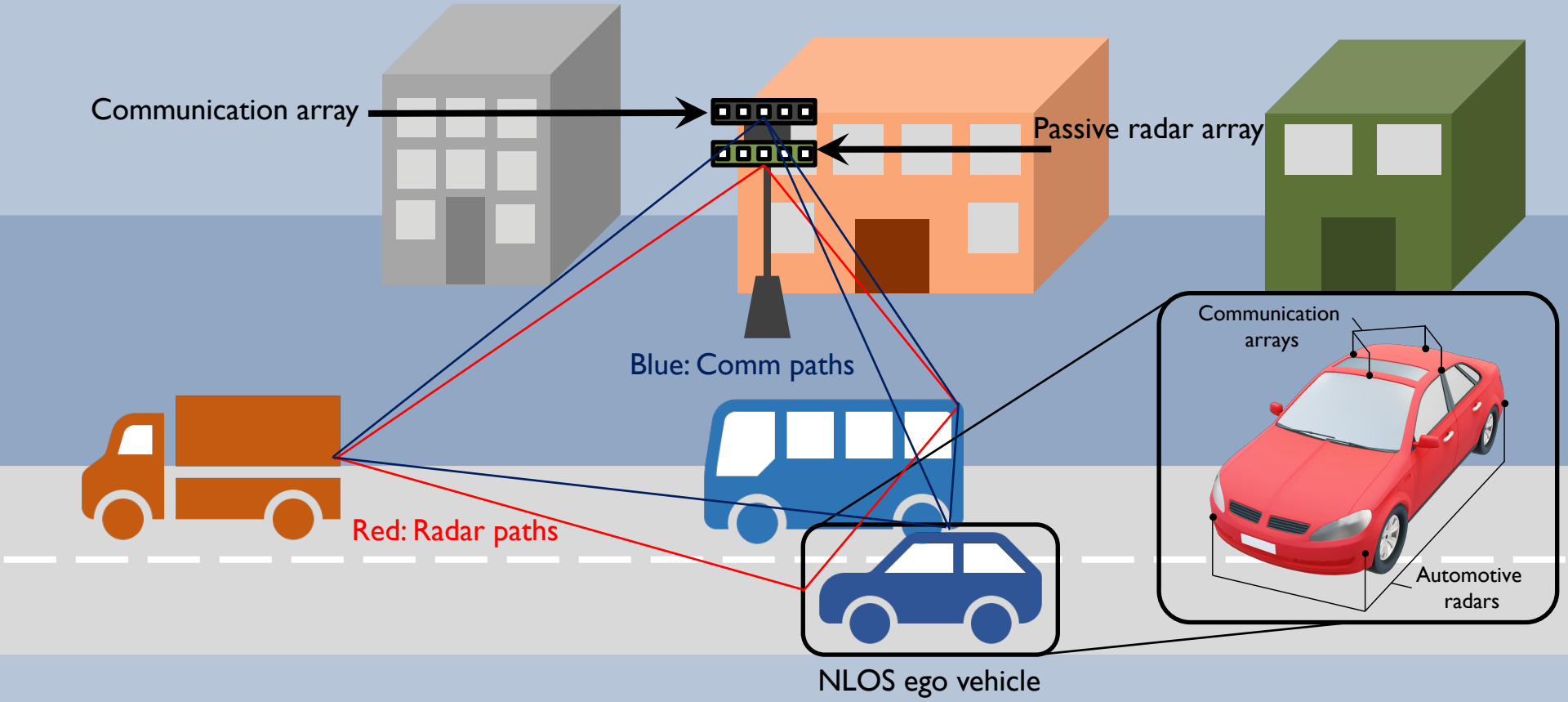
Contribution III

Millimeter wave link configuration using passive radar

A. Ali, N. Gonzalez-Prelcic, R. W. Heath Jr., and A. Ghosh, "Automotive radar as signals of opportunity for millimeter wave V2I links," ASILOMAR, 2019 (invited paper).

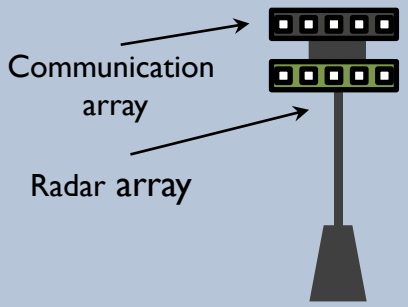
A. Ali, N. Gonzalez-Prelcic, R. W. Heath Jr., and A. Ghosh, "Passive Radar at the Roadside Unit to Configure Millimeter Wave V2I Links," under preparation.

Main idea: Passive radar at the RSU



Similarity expected in radar and comm. azimuth power spectrum (APS)

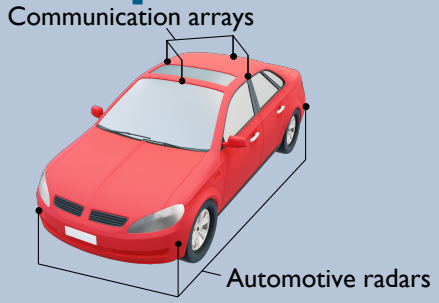
Communication and radar system setup



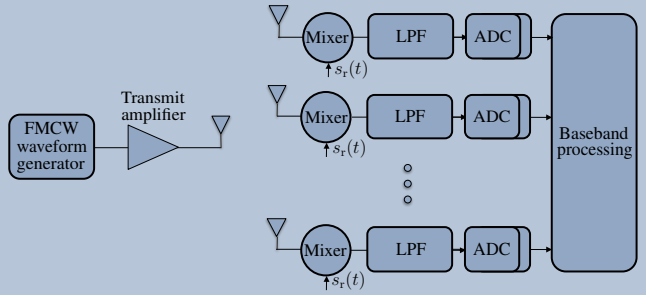
Co-located arrays at RSU, separated at the vehicle

FMCW radar and OFDM communication system

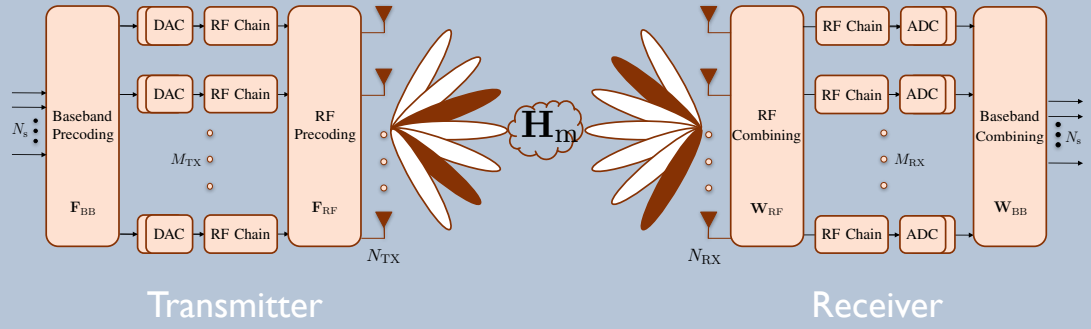
Same number of antennas in radar and communication array



Fully digital radar system



Hybrid mmWave system

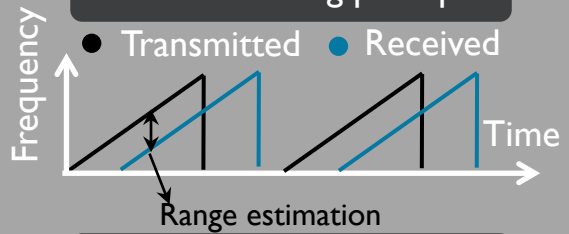


Estimate radar receive covariance and use it for mmWave link configuration

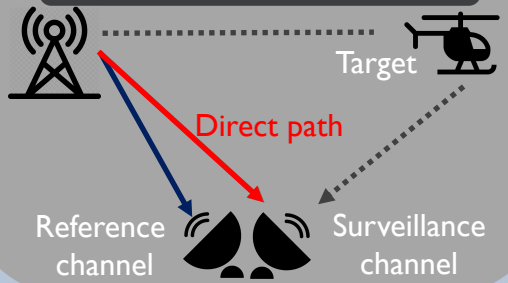
Challenges in using FMCW passive radar

Waveform knowledge in passive radar

FMCW working principle



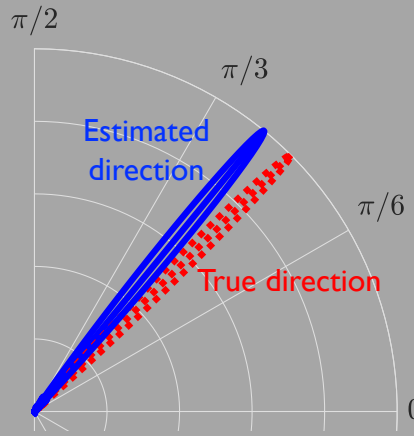
Passive radar



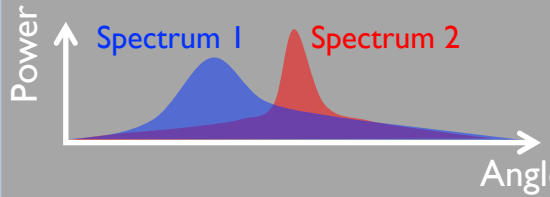
Bias in angle estimation [1]

Estimated angle $\hat{\theta} \neq \theta$ True angle

Critical in large antenna systems



Similarity metric



Are spectrum1 and 2 similar?

↓

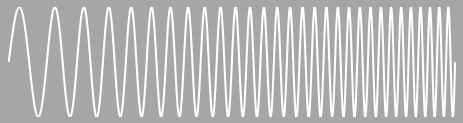
Relate spectrum similarity to a comm. metric e.g., rate?

Recover APS/covariance w/o waveform, correct bias, and measure similarity

[1] J. Kim, J. Chun, and S. Song, "Joint Range and Angle Estimation for FMCW MIMO Radar and Its Application," arXiv preprint arXiv:1811.06715, 2018.

Lack of waveform knowledge at roadside unit

Perfect FMCW waveform knowledge



Freq. of FMCW waveform

Initial frequency → $f_r + \frac{\beta}{2}t$ → Chirp rate

Freq. of processed signal

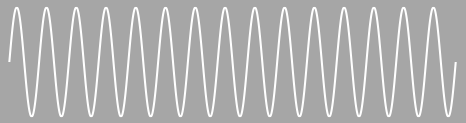
$\beta\tau$ → Used in range estimation

Phase of processed signal

$2\pi f_r \tau - \pi\beta\tau^2$

Used in Doppler estimation

Simple mixing



Proposed mixing

Freq. → $f_r + \Delta f$ Phase → ϵ

Freq. of processed signal

$\beta\tau + \Delta f - \frac{\beta t}{2}$ → Random

Phase of processed signal

$2\pi f_r \tau - \pi\beta\tau^2 + \epsilon$

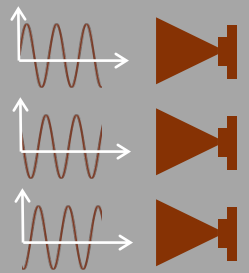
↓

Random

Spatial covariance

No time dependence

Depends only on phase-difference across antennas



Proposed architecture has same spatial covariance as perfect waveform

Proposed architecture recovers spatial cov. perfectly (not range or doppler)

Bias correction for FMCW radar

Bias in FMCW

$$\sin\hat{\theta} = \left(1 + \frac{B_\mathrm{r}}{2f_\mathrm{r}}\right)\sin\theta$$

Estimated angle — True angle

Particularly problematic in large antenna systems

Example: 256 antennas

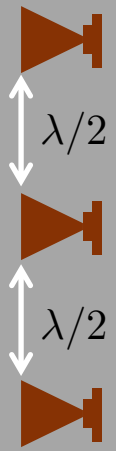
$f_\mathrm{r} = 76\,\mathrm{GHz}$

$B_\mathrm{r} = 1.2\,\mathrm{GHz}$

Possibly null in the direction of true angle

Bias in frequency division duplex systems

Same array in uplink and downlink



Array set to downlink wavelength, observations made in uplink

$$\sin\hat{\theta} = \frac{\lambda_\mathrm{DL}}{\lambda_\mathrm{UL}}\sin\theta$$

DL wavelength

UL wavelength

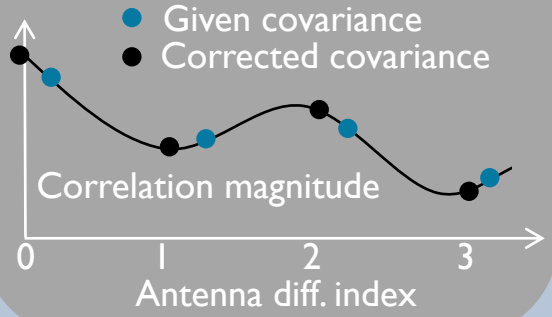
Similar bias in FDD

Covariance correction to rectify bias

Why covariance?

Alternate uses (precoding based on singular vectors)

A cov. correction strategy [1]



Established a connection with a well studied problem to correct the bias

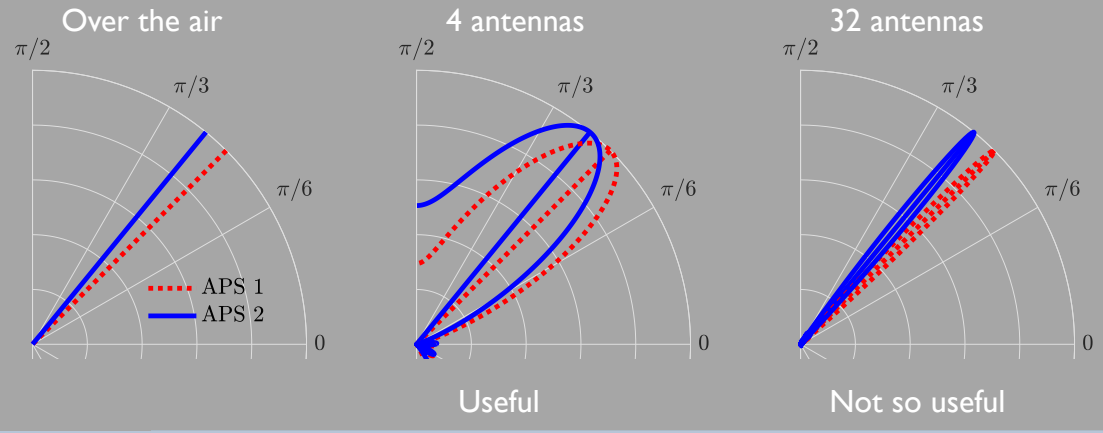
[1] M. Jordan, X. Gong, and G. Ascheid, "Conversion of the spatio-temporal correlation from uplink to downlink in FDD systems," in Proc. IEEE WCNC, Apr. 2009, pp. 1–6.

Similarity metric to compare power spectra

Why a similarity metric?

- Intuitive understanding of congruence but not formal
- A metric that informs about the system performance
- Useful also in sub-6 GHz aided mmWave and FDD

Role of system dimension



Need a similarity metric for spectra that informs about system performance

Proposed similarity metric

Definition

For two N point spectra $\mathbf{d}_1$ and $\mathbf{d}_2$

$$S_{1\to 2}(L,N)=\frac{\sum_{i\in\mathcal{I}_1}\mathbf{d}_2[i]}{\sum_{i\in\mathcal{I}_2}\mathbf{d}_2[i]}$$

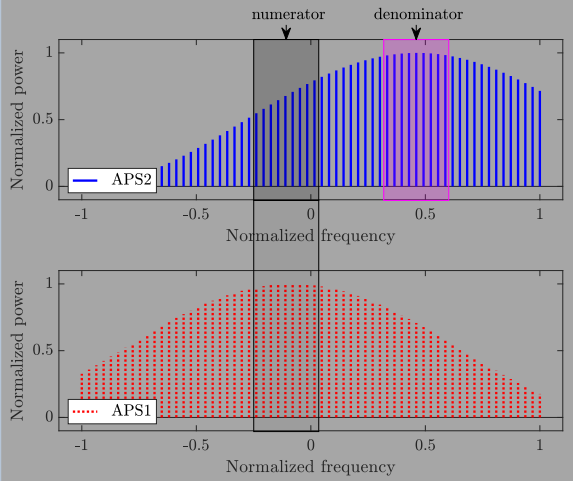
$\mathcal{I}_1$ $(\mathcal{I}_2)$ is the index set of cardinality $L \leq N$

Consists of indices of L largest entries of $\mathbf{d}_1$ $(\mathbf{d}_2)$

N related to antennas, L to streams

S normalized in [0,1]

Intuitive explanation



Generally sim. increases (decreases) with L (N)

Relation with rate

Similarity related with RPE

$$\mathrm{RPE}=\frac{\mathrm{tr}(\mathbf{U}_1^*\mathbf{R}_2\mathbf{U}_1)}{\mathrm{tr}(\mathbf{U}_2^*\mathbf{R}_2\mathbf{U}_2)}$$

Dominant singular vectors of R1

Dominant singular vectors of R2

Similarity identical to RPE

Angles on DFT grid or infinite antennas

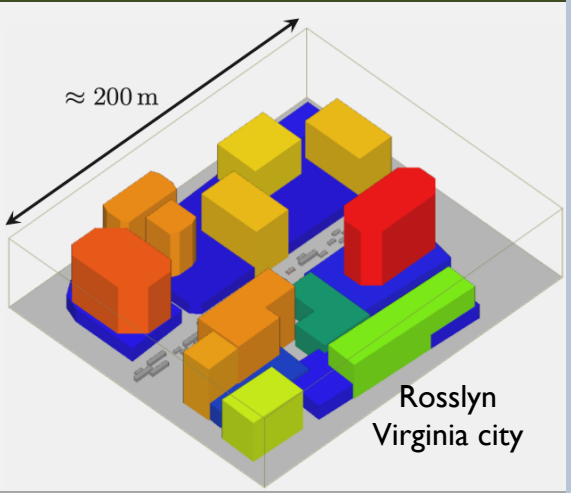
RPE related with rate [1]

The proposed similarity metric for power spectra informs about relative rate

[1] S. Park et. al., ``Spatial Channel Covariance Estimation for Hybrid Architectures Based on Tensor Decompositions", submitted to IEEE Trans. Wireless Commun.

Ray-tracing simulation scenario

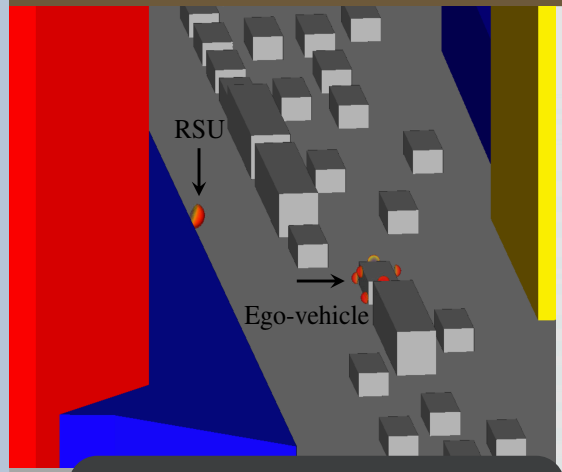
City, buildings, and vehicle position



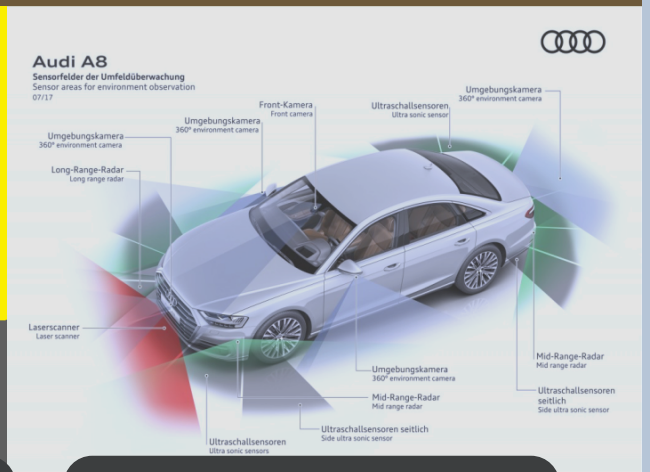
Cars and trucks in 4 lanes

Lane widths, car and truck ratio, inter-vehicle distance, lane speeds [1]

Communication and radar



Location of communication antennas on vehicle and RSU height [1]

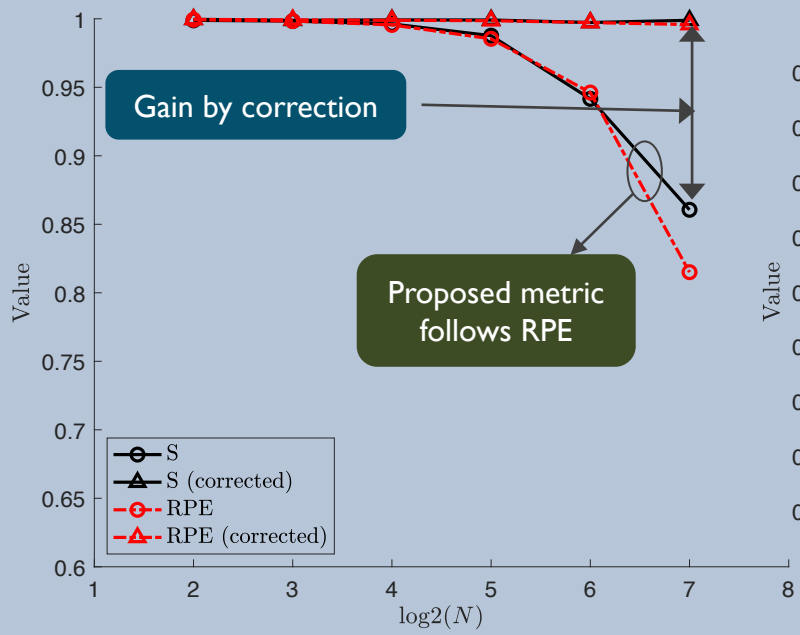


Position of radars not standardized
Used Audi A8 as an example

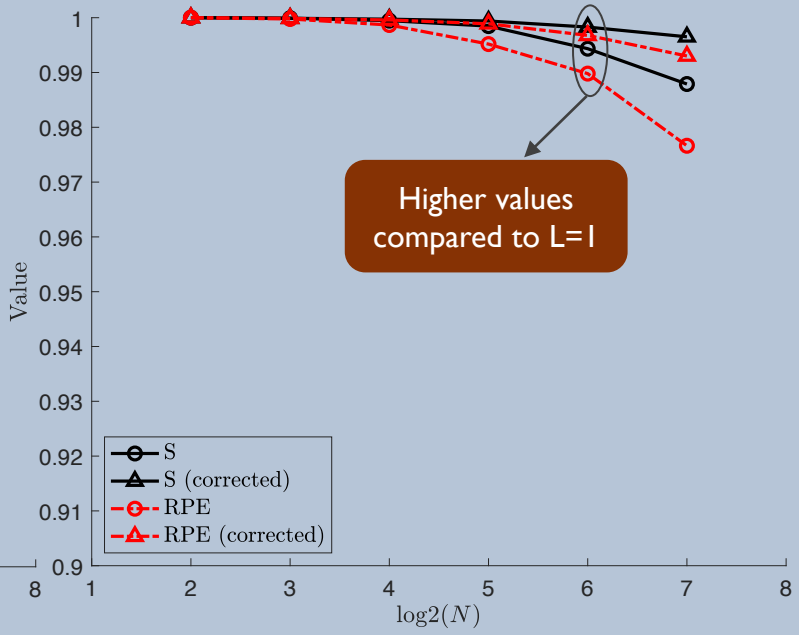
Ray-tracing simulation consistent with 3GPP-V2X evaluation methodology

[1] 3GPP, “Study on evaluation methodology of new Vehicle-to-Everything V2X use cases for LTE and NR,” 3GPP, TR 37.885, Sep. 2018, version 15.1.0.

Simulation result: Bias correction



N_s=L=1



N_s=L=4

Purpose

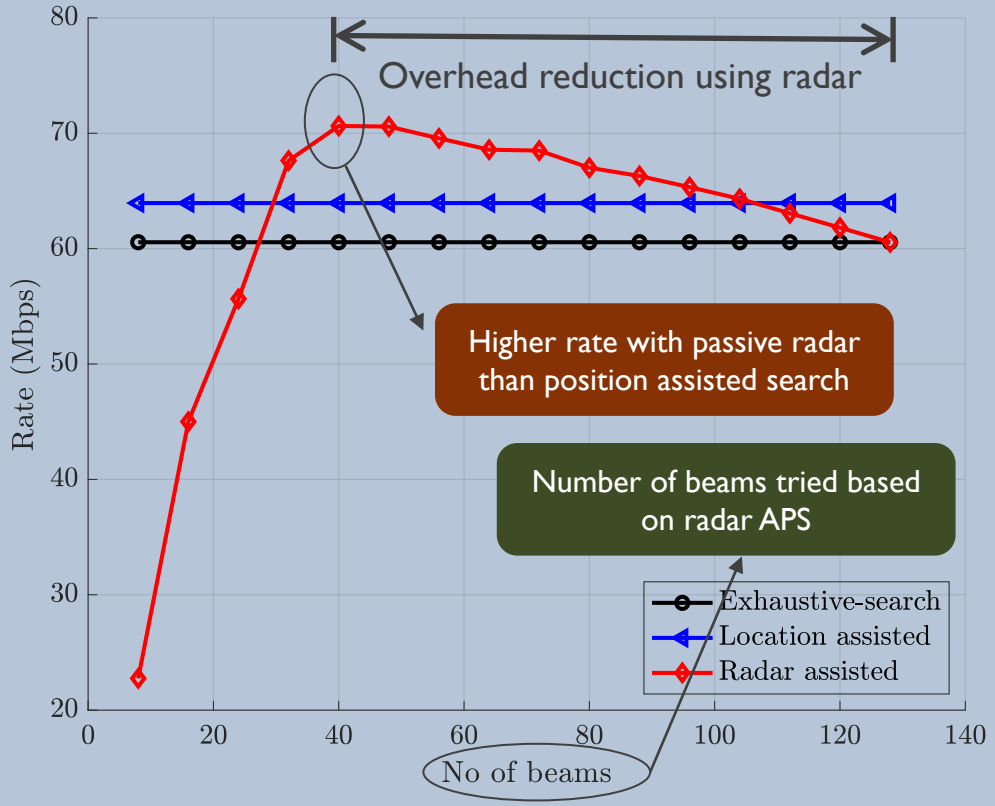
- RPE and similarity metric behavior for off-grid case
- Bias correction benefit
- Dependence on L and N

Simple set-up

- Co-located antennas
- Same frequency band

Bias correction improves the similarity between radar and communication

Simulation result: NLOS beam-selection



Parameter	Value
Radar frequency	76 GHz
Communication frequency	73 GHz
Antennas at RSU	128
Comm. antennas at Vehicle	16
Radar and Communication BW	1 GHz
GNSS error	10 m
Transmit power	30 dBm

Proposed radar assisted strategy particularly useful in NLOS scenario

Summary

Contributions

- Proposed a simplified radar receiver to recover spatial info.
- Proposed a radar bias correction strategy
- Proposed a similarity metric to compare power spectra

Results

- Bias correction improves the similarity between radar and communication covariance
- Passive radar assisted beam-selection has high rate in NLOS

Concluding remarks

Sub-6 GHz assisted mmWave

Possible spatial congruence but array size vastly different

Beam-training with reduced overhead

- Weighted compressed sensing
- Structured random codebook

Covariance estimation

- Parametric covariance translation
- Hermitian-sparse-weighted recovery

Radar assisted mmWave

Adjacent band but (i) signal processing challenges and (ii) non-adjacent locations

Beam-training in mmWave

- Simplified architecture
- Bias correction
- NLOS beam-search

Result: Out-of-band aided mmWave link configuration has a low training overhead in comparison with in-band only link configuration.

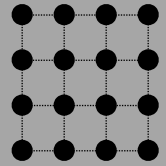
Future research directions

Experimental verification

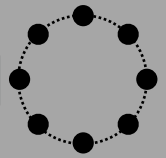
- Sub-6 GHz assisted: Measured channels / Prototype
- Radar assisted: Prototyping a passive radar based system

Array geometry

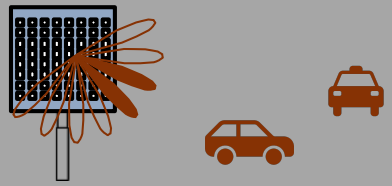
- Planar array



- Circular array

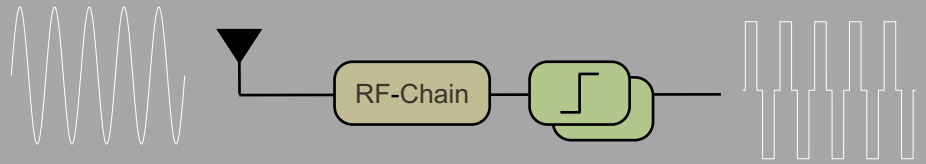


Multiuser scenario



- Overlapping signature of multiple vehicles

Low resolution architectures



Related publications

[1] A. Ali, N. Gonzalez-Prelcic, and R. W. Heath Jr, "Spatial Covariance Estimation for Millimeter Wave Hybrid Systems using Out-of-Band Information," IEEE Trans. Wireless Commun (early access).

[2] A. Ali, N. Gonzalez-Prelcic, and R. W. Heath Jr., "Millimeter wave beam-selection using out-of-band spatial information," IEEE Trans. Wireless Commun., vol. 17, no. 2, pp. 1038–1052, 2018.

[3] N. Gonzalez-Prelcic, A. Ali, V. Va and R. W. Heath Jr., "Millimeter Wave communication with out-of-band information", IEEE Commun. Mag., vol. 55, no. 12, pp. 140-146, 2017.

[4] A. Ali, N. Gonzalez-Prelcic, R. W. Heath Jr., and A. Ghosh, "Passive Radar at the Roadside Unit to Configure Millimeter Wave V2I Links," under preparation.

[5] A. Ali, N. Gonzalez-Prelcic, R. W. Heath Jr., and A. Ghosh, "Leveraging Sensing at the Infrastructure for mmWave Communication ," under preparation.

[6] A. Ali and R. W. Heath Jr., "Compressed beam-selection in millimeter wave systems with out-of-band partial support information," in Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP), Mar. 2017, pp. 3499–3503.

[7] A. Ali, N. Gonzalez-Prelcic, and R. W. Heath Jr., "Estimating millimeter wave channels using out-of-band measurements," in Proc. Inf. Theory Appl. (ITA) Wksp, Feb. 2016, pp. 1–5.

[8] A. Ali, N. Gonzalez-Prelcic, and A. Ghosh, "Millimeter wave V2I beam-training using base-station mounted radar," in Proc. IEEE Radar Conf., Apr. 2019, pp. 1-5.

[9] A. Ali, N. Gonzalez-Prelcic, R. W. Heath Jr., and A. Ghosh, "Automotive radar as signals of opportunity for millimeter wave V2I links," selected for presentation at Asilomar Conference, 2019.

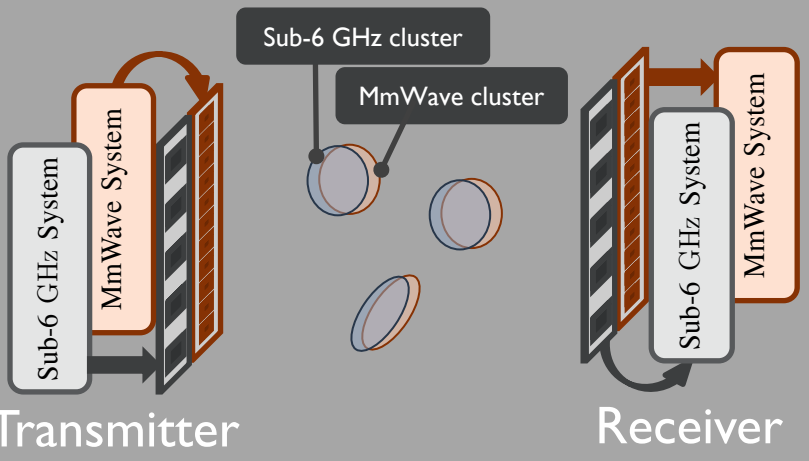
Thank you!

Multi-frequency channel models

Generate sub-6 GHz and mmWave channels for numerical evaluations

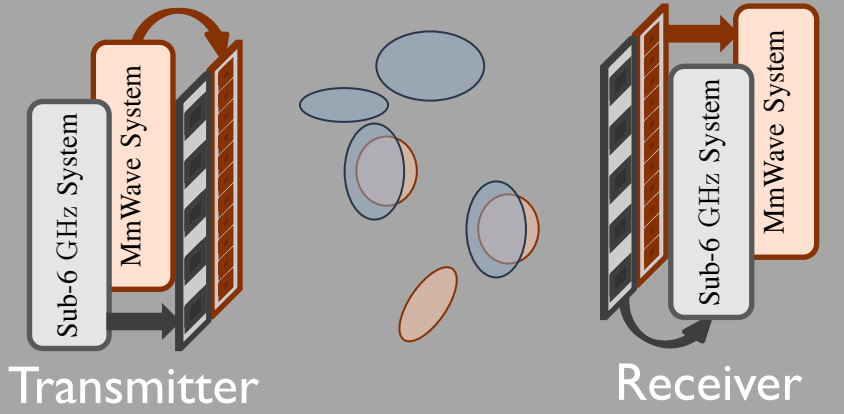
3GPP [1]

Equal number of clusters | Same mean angles | Same angle spreads



Proposed

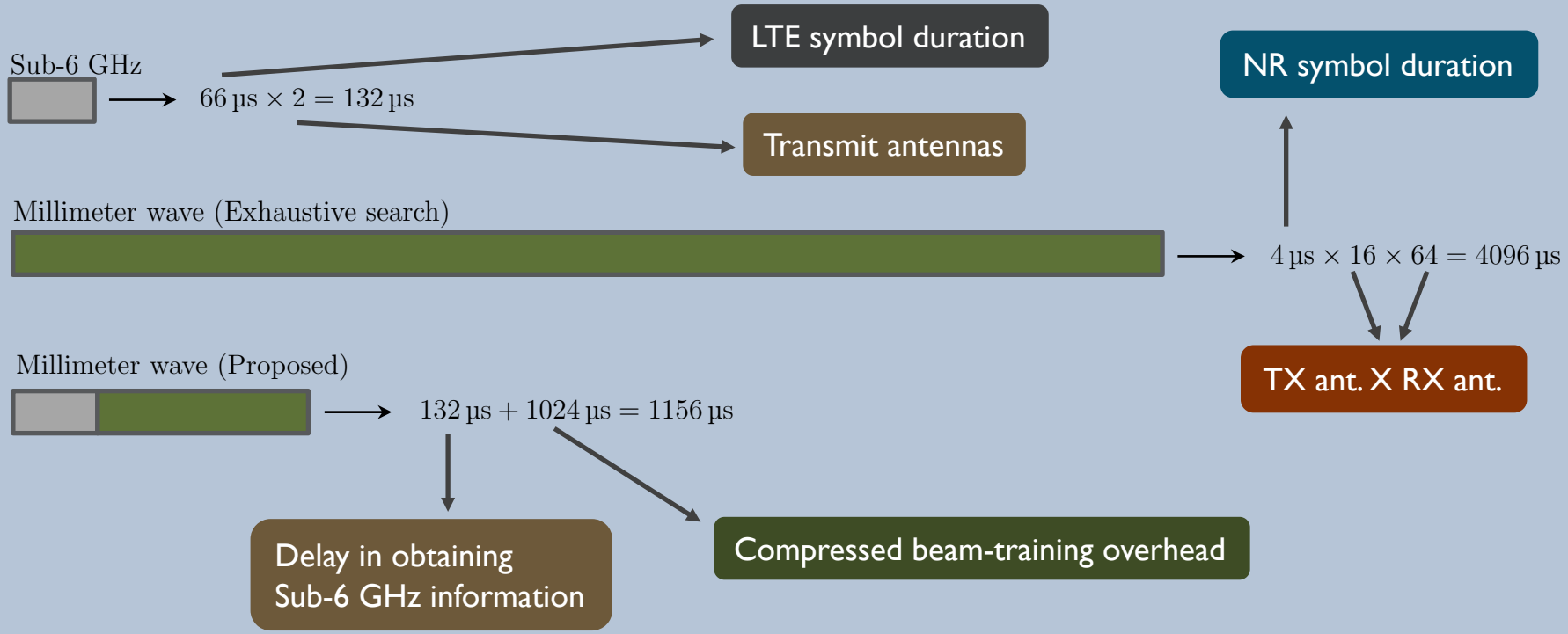
Different number of clusters | Different mean angles | Different angle spreads



A channel generation strategy consistent with empirical observations

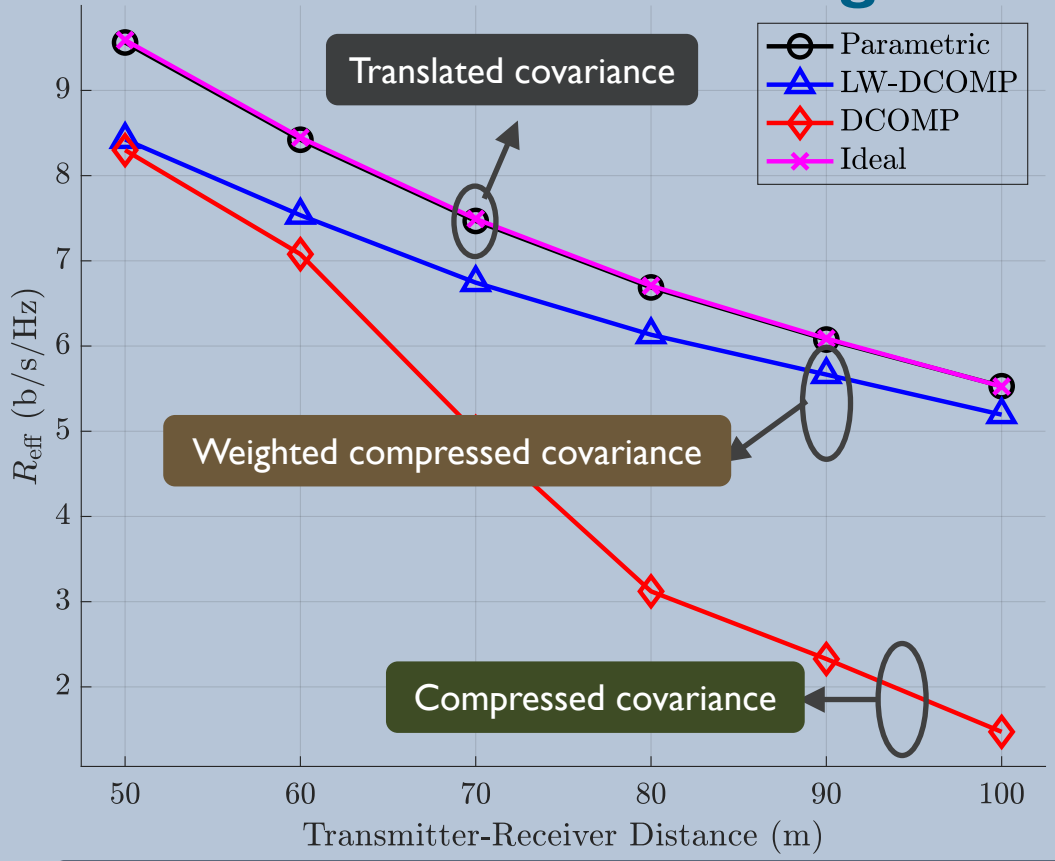
[1] 3GPP, "Study on channel model for frequencies from 0.5 to 100 GHz," 3rd Generation Partnership Project (3GPP), TR 38.901 V14.1.1, July. 2017.

Time in sub-6 GHz assisted beam-training



Sub-6 GHz assisted beam-training takes less time even with wait time

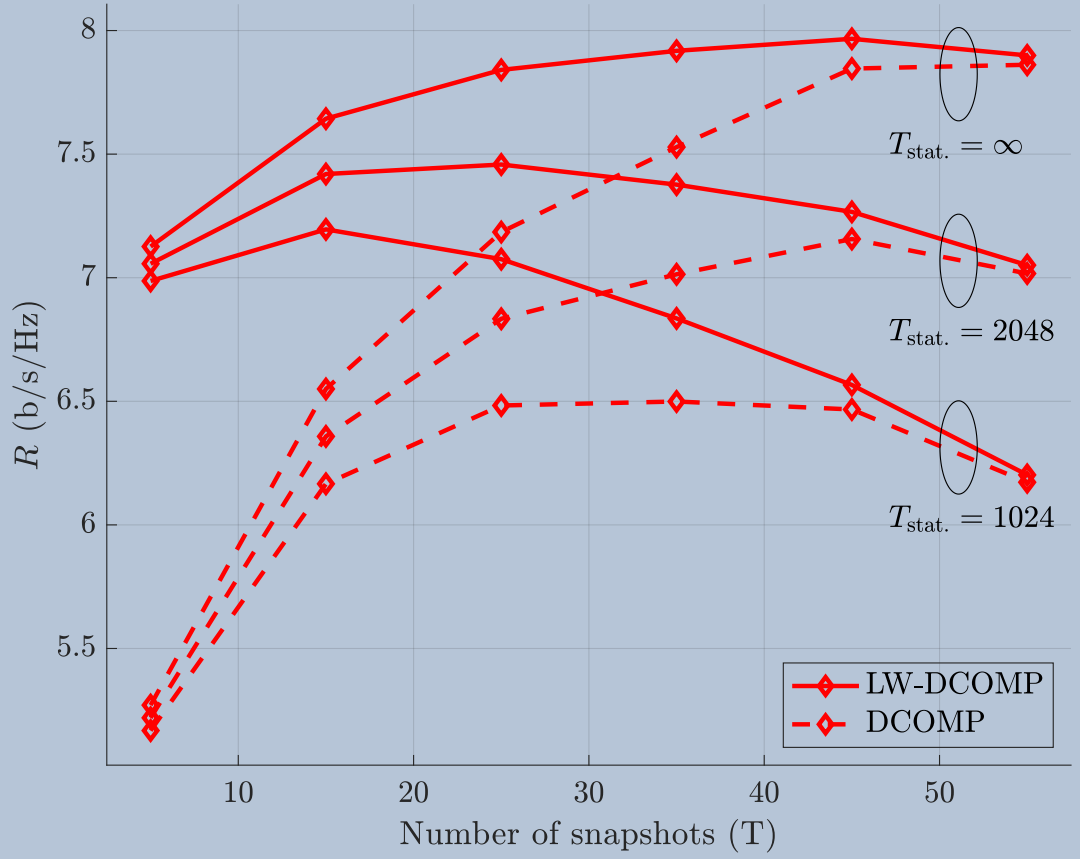
Simulation results: Single cluster with low spread



Parameter	Value
Sub-6 GHz frequency	3.5 GHz
Sub-6 GHz bandwidth	150 MHz
MmWave frequency	28 GHz
MmWave bandwidth	800 MHz
Sub-6 GHz (TX/RX) antennas	4/8
MmWave (TX/RX) antennas	32/64
MmWave (TX/RX) RF chains	8/16

Performance comparable to perfect covariance in limited scattering channels

Simulation results: Overhead for cov. estimation



Parameter	Value
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MmWave frequency	28 GHz
MmWave bandwidth	800 MHz
Sub-6 GHz (TX/RX) antennas	4/8
MmWave (TX/RX) antennas	32/64
MmWave (TX/RX) RF chains	8/16

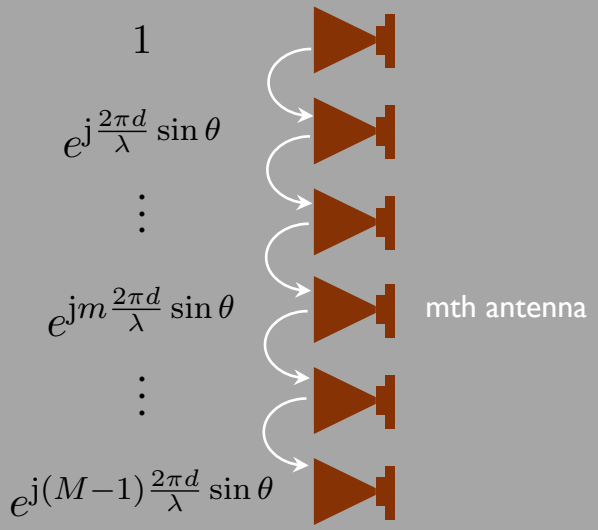
Potentially 9X overhead reduction using sub-6 GHz information

Loss in received SNR due to imperfect covariance

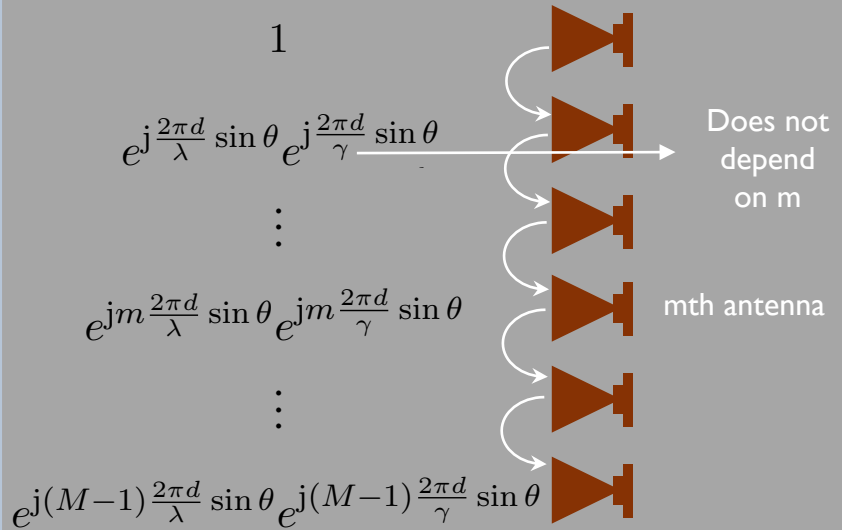
Error Model	$\hat{\mathbf{R}}_{\mathrm{RX}} = \mathbf{R}_{\mathrm{RX}} + \Delta\mathbf{R}_{\mathrm{RX}} \qquad \hat{\mathbf{R}}_{\mathrm{TX}} = \mathbf{R}_{\mathrm{TX}} + \Delta\mathbf{R}_{\mathrm{TX}}$
SNR loss	$\gamma = \dfrac{\mathrm{SNR}_{\mathbf{R}}}{\mathrm{SNR}_{\hat{\mathbf{R}}}}$
Bounds on loss	$\gamma \approx \left(1 + \dfrac{\lVert \Delta\mathbf{R}_{\mathrm{RX}}\mathbf{U}_{\mathrm{RX,s}} \rVert^2}{N_{\mathrm{RX}}^2\sigma_\alpha^4}\right)\left(1 + \dfrac{\lVert \Delta\mathbf{R}_{\mathrm{TX}}\mathbf{U}_{\mathrm{TX,s}} \rVert^2}{N_{\mathrm{TX}}^2\sigma_\alpha^4}\right)$
Insight	Loss proportional to the alignment of channel subspace with the perturbation subspace

Bias in FMCW radar

Angle dependent phase offset in comm.



Angle dependent phase offset in FMCW



Additional phase variation causes bias in FMCW radar