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Cockrell School of Engineering

Spatial channel covariance estimation for the hybrid architecture at a base station - A tensor-decomposition-based approach

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Overview

We proposed a ***spatial channel covariance*** estimation method

for ***hybrid*** analog/digital architecture

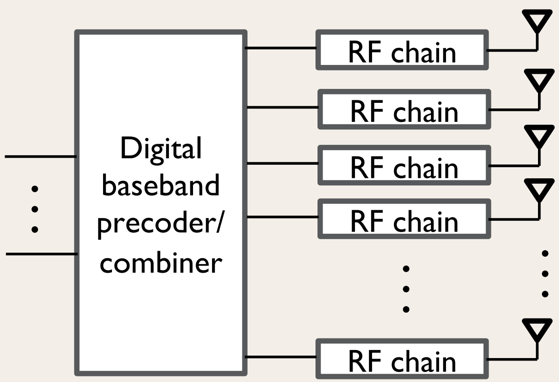
over ***spatially sparse frequency-selective*** channels

based on ***higher-order tensor decompositions***

Background

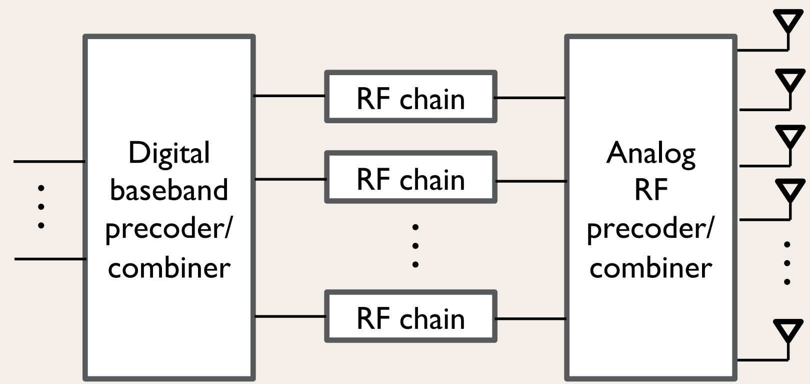
Hybrid analog/digital architecture

Fully digital architecture
(Conventional MIMO)



High complexity & power consumption due to ADC/DACs in RF chains

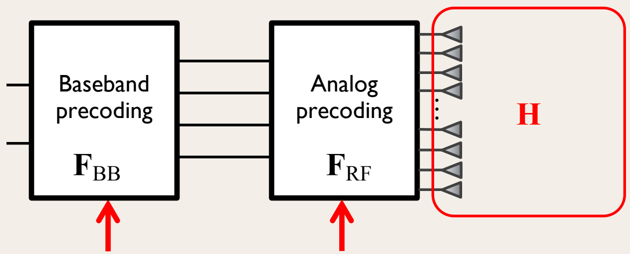
Hybrid analog/digital architecture



Compromise between complexity and performance

Spatial channel covariance information

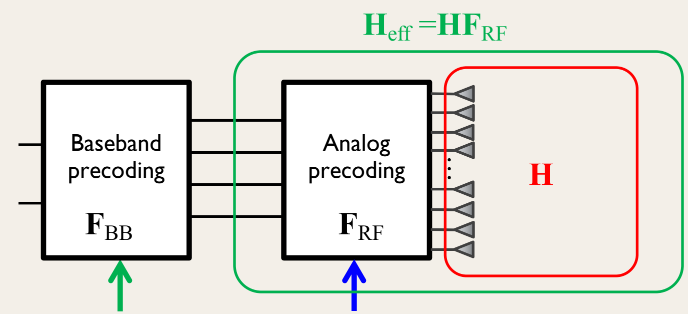
Hybrid precoding design based on *instantaneous full CSIT*



Instantaneous full CSI of the entire channel matrix $\mathbf{H}$

difficult to obtain over time-varying channels

Hybrid precoding design based on *spatial channel covariance* information



Instantaneous CSI of effective channel $\mathbf{H}_{\mathrm{eff}}$

Spatial channel covariance of $\mathbf{H}$ ($\mathrm{E}[\mathbf{H}\mathbf{H}^*]$)

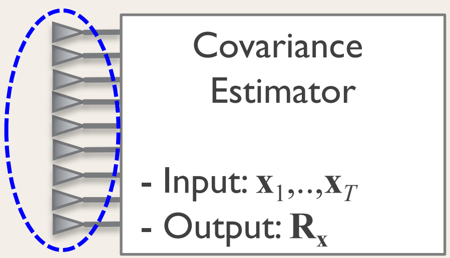
w/ reduced dimension

more feasible to estimate over time-varying channels

Both methods have similar spectral efficiency for spatially sparse channels

Is covariance estimation a challenging task?

Fully digital processing



$\mathbf{x}$

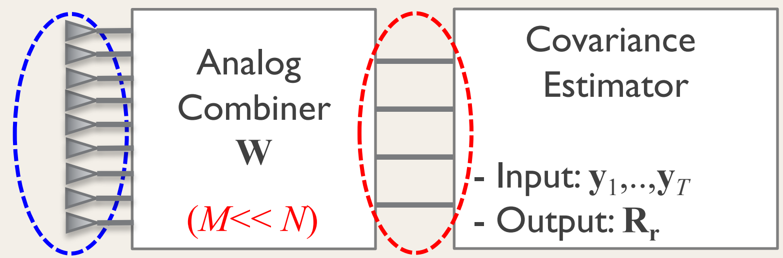
$(N \times 1)$

$$\mathbf{R}_\mathbf{x} = \mathbb{E}[\mathbf{x}\mathbf{x}^*] \approx \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t\mathbf{x}_t^*$$

Covariance estimation is not challenging

VS.

Hybrid analog/digital processing



$\mathbf{x}$ | $\mathbf{y}\ (= \mathbf{W}^*\mathbf{x})$

raw measurements ($N_{\mathrm{ant}} \times 1$ vectors)

low-dimensional projections ($M_{\mathrm{RF}} \times 1$ vectors)

How to estimate $\mathbf{R}_\mathbf{x}$ with only $\mathbf{y}_t$'s ?
→challenging

There are some prior work based on MUSIC/ESPIRT or compressive sensing

Signal model and motivation

Tensor signal model of wideband channel

Wideband channel vector at subcarrier k & frame t

antenna (N_{ant})

$$\mathbf{h}_{t,k} = \sum_{d=0}^{N_{\mathrm{CP}}-1} \mathbf{h}_t[d] e^{-\frac{j2\pi(k-1)d}{K_{\mathrm{sbcr}}}} \quad \text{where} \quad \mathbf{h}_t[d] = \sum_{\ell=1}^{L_{\mathrm{ch}}} g_{t,\ell}\, p_{\mathrm{PS}}(dT_s - \tau_\ell)\mathbf{a}(\phi_\ell)$$

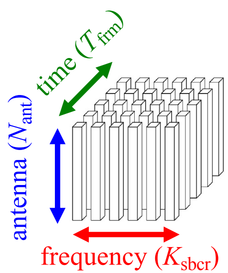
The wideband channel can be represented as the *CPD* *(Canonical polyadic decomposition)* form of a *3D-tensor*.

3D tensor signal model of wideband channel (antenna, subcarrier, frame)

- t: frame index
- k: subcarrier index
- l: channel path index
- d: delay tap index
- N_{CP}: cyclic prefix size
- K_{sbcr}: # of subcarriers
- T_{frm}: # of frames
- L_{ch}: # of channel paths
- $g_{t,l}$: channel path gain
- T_s: sampling period
- τ_l : channel path delay
- ϕ_l : angle-of-arrival (AoA)
- $p_{\mathrm{PS}}(t)$: analog filter

$$\mathcal{H} \in \mathbb{C}^{N_{\mathrm{ant}} \times K_{\mathrm{sbcr}} \times T_{\mathrm{frm}}} \quad \text{where} \quad [\mathcal{H}]_{n,k,t} = \sum_{\ell=1}^{L_{\mathrm{ch}}} a_{n,\ell} c_{k,\ell} g_{t,\ell}$$

Three factor matrices



$$a_{n,\ell} = [\mathbf{A}]_{n,\ell}$$

$$\mathbf{A} = \begin{bmatrix}\mathbf{a}_1 & \cdots & \mathbf{a}_{L_{\mathrm{ch}}}\end{bmatrix} = \begin{bmatrix}\mathbf{a}(\phi_1) & \cdots & \mathbf{a}(\phi_{L_{\mathrm{ch}}})\end{bmatrix}$$

mode-1 factor matrix ($N_{\mathrm{ant}} \times L_{\mathrm{ch}}$)

Space domain (antenna)

$$c_{k,\ell} = [\mathbf{C}]_{k,\ell}$$

$$\mathbf{C} = \begin{bmatrix}\mathbf{c}_1 & \cdots & \mathbf{c}_{L_{\mathrm{ch}}}\end{bmatrix} = \begin{bmatrix} c_{1,1} & \cdots & c_{1,L_{\mathrm{ch}}} \\ \vdots & \ddots & \vdots \\ c_{K_{\mathrm{sbcr}},1} & \cdots & c_{K_{\mathrm{sbcr}},L_{\mathrm{ch}}} \end{bmatrix}$$

$$c_{k,\ell} = \sum_{d=0}^{N_{\mathrm{CP}}-1} p_{\mathrm{PS}}(dT_s - \tau_\ell) e^{-\frac{j2\pi(k-1)d}{K_{\mathrm{sbcr}}}}$$

mode-2 factor matrix ($K_{\mathrm{sbcr}} \times L_{\mathrm{ch}}$)

Frequency domain (subcarrier)

$$g_{t,\ell} = [\mathbf{G}]_{t,\ell}$$

$$\mathbf{G} = \begin{bmatrix}\mathbf{g}_1 & \cdots & \mathbf{g}_{L_{\mathrm{ch}}}\end{bmatrix} = \begin{bmatrix} g_{1,1} & \cdots & g_{1,L_{\mathrm{ch}}} \\ \vdots & \ddots & \vdots \\ g_{T_{\mathrm{frm}},1} & \cdots & g_{T_{\mathrm{frm}},L_{\mathrm{ch}}} \end{bmatrix}$$

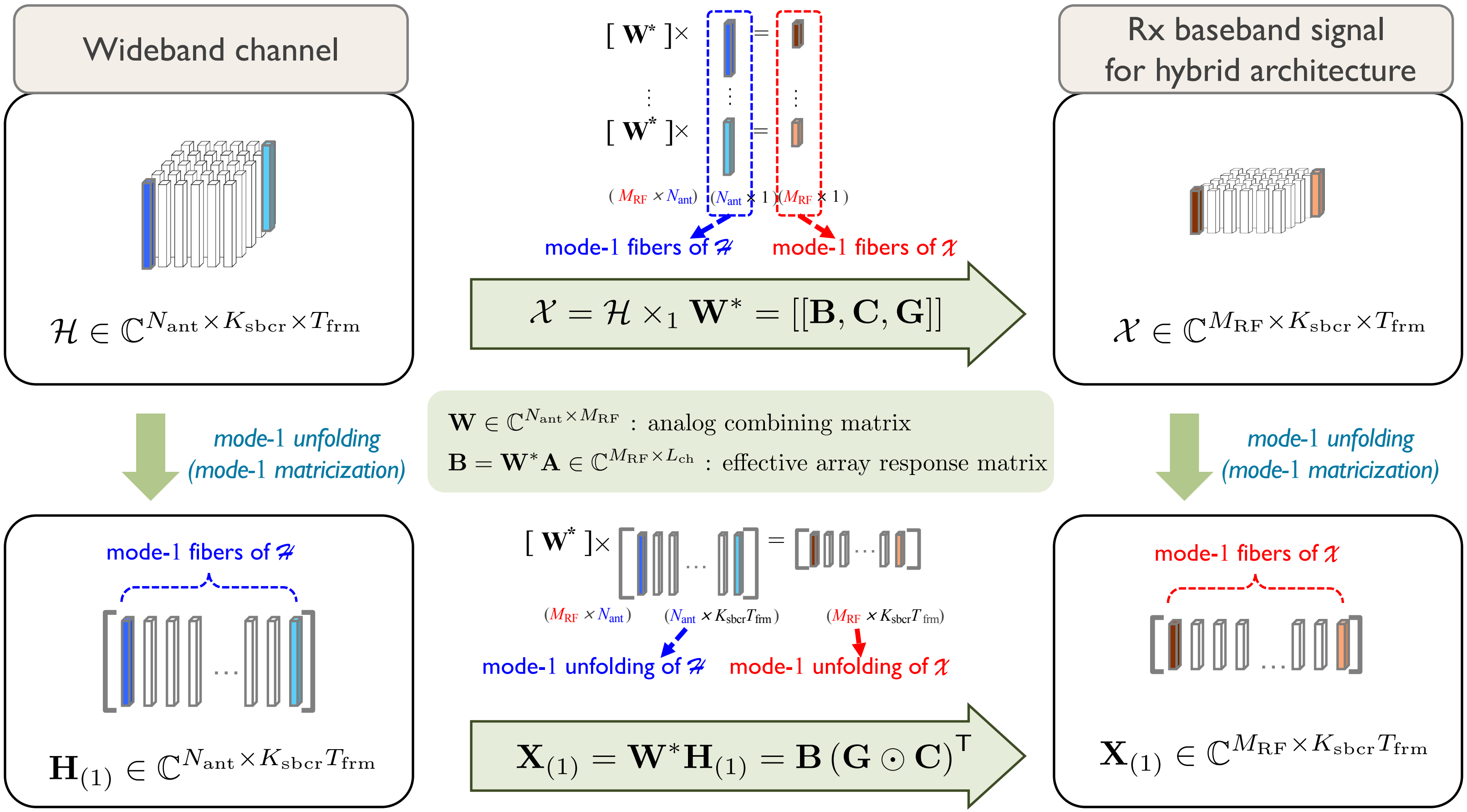
mode-3 factor matrix ($T_{\mathrm{frm}} \times L_{\mathrm{ch}}$)

Time domain (frame)

CPD (Canonical polyadic decomposition)

CPD	Factorizing a tensor into a sum of component rank-one tensors $\mathcal{X} = \sum_{\ell=1}^{L_{\text{ch}}} \underbrace{\mathbf{b}_\ell \circ \mathbf{c}_\ell \circ \mathbf{g}_\ell}_{\text{rank-one tensor}} \Leftrightarrow [\mathcal{X}]_{m,k,t} = \sum_{\ell=1}^{L_{\text{ch}}} b_{m,\ell} c_{k,\ell} g_{t,\ell}$ $\circ$: outer product L_{ch} → *tensor rank*
Factor matrices	Combination of the vectors from the rank-one components $\mathbf{B} = [\mathbf{b}_1 \ \cdots \ \mathbf{b}_{L_{\text{ch}}}]$ $\mathbf{C} = [\mathbf{c}_1 \ \cdots \ \mathbf{c}_{L_{\text{ch}}}]$ $\mathbf{G} = [\mathbf{g}_1 \ \cdots \ \mathbf{g}_{L_{\text{ch}}}]$ → *All factor matrices have L_{ch} columns.*
Uniqueness of CPD	If a ***higher-order*** tensor has a ***low tensor rank***, its ***CPD is unique*** under some mild constraints. $\mathcal{X} = \sum_{\ell=1}^{L_{\text{ch}}} \mathbf{b}_\ell \circ \mathbf{c}_\ell \circ \mathbf{g}_\ell$ → *This is the only possible combination of rank-one tensors that sums to the given tensor with the exception of two types of **indeterminacy**: **scaling** and **permutation**.* 1) *Scaling indeterminacy* : $\mathcal{X} = [[\mathbf{B}, \mathbf{C}, \mathbf{G}]] \Leftrightarrow \mathcal{X} = [[\mathbf{B}\boldsymbol{\Delta}_\mathbf{B}, \mathbf{C}\boldsymbol{\Delta}_\mathbf{C}, \mathbf{G}\boldsymbol{\Delta}_\mathbf{G}]]$ for any diagonal matrices satisfying $\boldsymbol{\Delta}_\mathbf{B}\boldsymbol{\Delta}_\mathbf{C}\boldsymbol{\Delta}_\mathbf{G} = \mathbf{I}$ 2) *Permutation indeterminacy* : $\mathcal{X} = [[\mathbf{B}, \mathbf{C}, \mathbf{G}]] \Leftrightarrow \mathcal{X} = [[\mathbf{B}\boldsymbol{\Pi}, \mathbf{C}\boldsymbol{\Pi}, \mathbf{G}\boldsymbol{\Pi}]]$ for any permutation matrix $\boldsymbol{\Pi}$

Tensor signal model for hybrid architecture



Proposed method

Overall framework

Goal: Estimate the *spatial channel covariance* matrix $\mathbf{R_h}$ for a given *measurement* tensor $\mathcal{X}$

input

Measurements at baseband

$\mathcal{X}$

$\mathcal{X} \in \mathbb{C}^{M_{\mathrm{RF}} \times K_{\mathrm{sbcr}} \times T_{\mathrm{frm}}}$

Relationship between $\mathcal{H}$ and $\mathcal{X}$ in hybrid architectures

$\mathcal{H} = [[\mathbf{A}, \mathbf{C}, \mathbf{G}]]$

$\mathcal{H} \in \mathbb{C}^{N_{\mathrm{ant}} \times K_{\mathrm{sbcr}} \times T_{\mathrm{frm}}}$

analog combiner $\mathbf{W}^*$ $(M_{\mathrm{RF}} \times N_{\mathrm{ant}})$

$\mathcal{X} = [[\mathbf{B}, \mathbf{C}, \mathbf{G}]]$

$\mathbf{B} = \mathbf{W}^* \mathbf{A}$

output

Spatial channel covariance matrix

$$\mathbf{R_h} = \frac{1}{K_{\mathrm{sbcr}} T_{\mathrm{frm}}} \sum_{k=1}^{K_{\mathrm{sbcr}}} \sum_{t=1}^{T_{\mathrm{frm}}} \mathcal{H}(:,k,t)\mathcal{H}(:,k,t)^*$$

$$= \frac{1}{K_{\mathrm{sbcr}} T_{\mathrm{frm}}} \mathbf{A} \left(\mathbf{G}^*\mathbf{G} \circledcirc \mathbf{C}^*\mathbf{C} \right) \mathbf{A}^*$$

Step 1. CPD of $\mathcal{X}$

sub-goal: *input* $\mathcal{X}$ → *output* $\{\hat{\mathbf{B}}, \hat{\mathbf{C}}, \hat{\mathbf{G}}\}$

Relationship between $\{\mathbf{B}, \mathbf{C}, \mathbf{G}\}$ and $\{\hat{\mathbf{B}}, \hat{\mathbf{C}}, \hat{\mathbf{G}}\}$

$$\hat{\mathbf{B}} \approx \mathbf{B}\mathbf{\Pi}\mathbf{\Delta_B}$$
$$\hat{\mathbf{C}} \approx \mathbf{C}\mathbf{\Pi}\mathbf{\Delta_C}$$
$$\hat{\mathbf{G}} \approx \mathbf{G}\mathbf{\Pi}\mathbf{\Delta_G}$$

actual factor matrices of $\mathcal{X}$ — *permutation matrix* — *diagonal matrices satisfying $\mathbf{\Delta_B}\mathbf{\Delta_C}\mathbf{\Delta_G} = \mathbf{I}$*

→ Both permutation and scaling indeterminacy

Step 2. Estimation of $\mathbf{A}$

sub-goal: *input* $\hat{\mathbf{B}}$ → *output* $\{\hat{\mathbf{A}}, \hat{\mathbf{\Delta}}_{\mathbf{B}}\}$

Relationship between $\mathbf{A}$ and $\hat{\mathbf{A}}$

$$\hat{\mathbf{A}} \approx \mathbf{A}\mathbf{\Pi}$$

actual factor matrix of $\mathcal{H}$ — *permutation matrix*

→ Only permutation indeterminacy

Step 3. Calculation of $\mathbf{R_h}$

sub-goal: *input* $\{\hat{\mathbf{A}}, \hat{\mathbf{\Delta}}_{\mathbf{B}}, \hat{\mathbf{C}}, \hat{\mathbf{G}}\}$ → *output* $\hat{\mathbf{R}}_{\mathbf{h}}$

Relationship between $\mathbf{R_h}$ and $\hat{\mathbf{R}}_{\mathbf{h}}$

$$\hat{\mathbf{R}}_{\mathbf{h}} \approx \mathbf{R_h}$$

Actual spatial channel covariance matrix

→ No indeterminacy

Step 1. CPD by ALS (Alternating Least Squares)

Goal	Estimate $\{\hat{\mathbf{B}}, \hat{\mathbf{C}}, \hat{\mathbf{G}}\}$ for a given $\mathcal{X}$
Problem formulation	$\{\hat{\mathbf{B}}, \hat{\mathbf{C}}, \hat{\mathbf{G}}\} = \arg\min_{\mathring{\mathbf{B}},\mathring{\mathbf{C}},\mathring{\mathbf{G}}} \left\| \mathcal{X} - [[\mathring{\mathbf{B}}, \mathring{\mathbf{C}}, \mathring{\mathbf{G}}]] \right\|$

** Equivalent form (using the Frobenius norms of mode-n unfolding matrices)*

$$\begin{aligned} \left\| \mathcal{X} - [[\mathring{\mathbf{B}}, \mathring{\mathbf{C}}, \mathring{\mathbf{G}}]] \right\| &= \|\mathbf{X}_{(1)} - \mathring{\mathbf{B}}(\mathring{\mathbf{G}} \odot \mathring{\mathbf{C}})^{\mathsf{T}}\|_F \\ &= \|\mathbf{X}_{(2)} - \mathring{\mathbf{C}}(\mathring{\mathbf{G}} \odot \mathring{\mathbf{B}})^{\mathsf{T}}\|_F \\ &= \|\mathbf{X}_{(3)} - \mathring{\mathbf{G}}(\mathring{\mathbf{C}} \odot \mathring{\mathbf{B}})^{\mathsf{T}}\|_F \end{aligned}$$

Solution (by using ALS)

Repeat

Sub-step 1. Fix $\mathring{\mathbf{C}}$ and $\mathring{\mathbf{G}}$. Then, update $\mathring{\mathbf{B}} \leftarrow \arg\min_{\mathring{\mathbf{B}}} \|\mathbf{X}_{(1)} - \mathring{\mathbf{B}}(\mathring{\mathbf{G}} \odot \mathring{\mathbf{C}})^{\mathsf{T}}\|_F$

Sub-step 2. Fix $\mathring{\mathbf{B}}$ and $\mathring{\mathbf{G}}$. Then, update $\mathring{\mathbf{C}} \leftarrow \arg\min_{\mathring{\mathbf{C}}} \|\mathbf{X}_{(2)} - \mathring{\mathbf{C}}(\mathring{\mathbf{G}} \odot \mathring{\mathbf{B}})^{\mathsf{T}}\|_F$

Sub-step 3. Fix $\mathring{\mathbf{B}}$ and $\mathring{\mathbf{C}}$. Then, update $\mathring{\mathbf{G}} \leftarrow \arg\min_{\mathring{\mathbf{G}}} \|\mathbf{X}_{(3)} - \mathring{\mathbf{G}}(\mathring{\mathbf{C}} \odot \mathring{\mathbf{B}})^{\mathsf{T}}\|_F$

** Solution of sub-step 1*

$$\begin{aligned} \mathring{\mathbf{B}} &= \mathbf{X}_{(1)} \left(\left(\mathring{\mathbf{G}} \odot \mathring{\mathbf{C}} \right)^{\mathsf{T}} \right)^{\dagger} \\ &= \mathbf{X}_{(1)} \left(\left(\mathring{\mathbf{G}} \odot \mathring{\mathbf{C}} \right) \left(\mathring{\mathbf{G}}^{*}\mathring{\mathbf{G}} \circledast \mathring{\mathbf{C}}^{*}\mathring{\mathbf{C}} \right)^{\dagger} \right)^{\mathsf{C}} \end{aligned}$$

Note. ALS ***guarantees*** convergence to a local (not global) optimal solution

Step 2. Estimation of $\mathbf{A}$

Goal	Estimate $\{\hat{\mathbf{A}}, \hat{\mathbf{\Delta}}_{\mathbf{B}}\}$ for a given $\hat{\mathbf{B}}$
Problem formulation	$\{\hat{\phi}_\ell, \hat{\delta}_{\mathbf{B},\ell}\} = \arg\min_{\phi,\delta} \lVert \hat{\mathbf{b}}_\ell - \mathbf{W}^*\mathbf{a}(\phi)\delta \rVert^2$ → *Find ϕ that minimizes the vector angle between $\hat{\mathbf{b}}_\ell$ and $\mathbf{W}^*\mathbf{a}(\phi)$*

$\mathbf{A} \in \mathbb{C}^{N_{\text{ant}} \times L_{\text{ch}}}$: The $N_{\text{ant}} L_{\text{ch}}$ complex-valued elements are determined by L_{ch} real-valued variables

→ structured matrix

$\mathbf{a}(\phi)$: array response vector

Solution (by using a correlation-based method)

$$\hat{\phi}_\ell = \arg\max_{\phi} \frac{|\hat{\mathbf{b}}_\ell^* \mathbf{W}^* \mathbf{a}(\phi)|}{\lVert \hat{\mathbf{b}}_\ell \rVert \lVert \mathbf{W}^* \mathbf{a}(\phi) \rVert} \Rightarrow \hat{\mathbf{A}} = \left[\mathbf{a}(\hat{\phi}_1) \quad \cdots \quad \mathbf{a}(\hat{\phi}_{L_{\text{ch}}})\right]$$

One-dimensional search

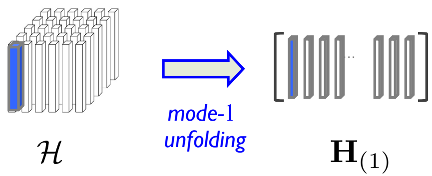
$$\hat{\delta}_{\mathbf{B},\ell} = \frac{\mathbf{a}^*(\hat{\phi}_\ell)\mathbf{W}\hat{\mathbf{b}}_\ell}{\lVert \mathbf{W}^*\mathbf{a}(\hat{\phi}_\ell) \rVert^2} \Rightarrow \hat{\mathbf{\Delta}}_{\mathbf{B}} = \operatorname{diag}\left(\left[\hat{\delta}_{\mathbf{B},1} \quad \cdots \quad \hat{\delta}_{\mathbf{B},L_{\text{ch}}}\right]\right)$$

Scaling-related information

Step 3. Calculation of spatial channel covariance

Goal	Estimate $\hat{\mathbf{R}}_{\mathbf{h}}$ for a given $\{\hat{\mathbf{A}}, \hat{\mathbf{\Delta}}_{\mathbf{B}}, \hat{\mathbf{C}}, \hat{\mathbf{G}}\}$

Problem formulation



- *mode-1* : space (antenna) domain
- *mode-2* : frequency (subcarrier) domain
- *mode-3* : time (frame) domain

$$\mathbf{R}_{\mathbf{h}} = \frac{1}{K_{\mathrm{sbcr}}T_{\mathrm{frm}}}\mathbf{H}_{(1)}\mathbf{H}_{(1)}^{*} = \frac{1}{K_{\mathrm{sbcr}}T_{\mathrm{frm}}}\mathbf{A}\left(\mathbf{G}^{*}\mathbf{G} \odot \mathbf{C}^{*}\mathbf{C}\right)\mathbf{A}^{*}$$

$$\mathbf{A} \approx \hat{\mathbf{A}}\mathbf{\Pi}^{*} \qquad \mathbf{C} \approx \hat{\mathbf{C}}\mathbf{\Delta}_{\mathbf{C}}^{-1}\mathbf{\Pi}^{*} \qquad \mathbf{G} \approx \hat{\mathbf{G}}\mathbf{\Delta}_{\mathbf{G}}^{-1}\mathbf{\Pi}^{*}$$

While $\{\hat{\mathbf{A}}, \hat{\mathbf{C}}, \hat{\mathbf{G}}\}$ are known, $\{\mathbf{\Pi}, \mathbf{\Delta}_{\mathbf{C}}, \mathbf{\Delta}_{\mathbf{G}}\}$ are unknown.

How to estimate $\mathbf{R}_{\mathbf{h}}$?

Calculation

$\mathbf{A} \approx \hat{\mathbf{A}}\mathbf{\Pi}^{*}$
$\mathbf{C} \approx \hat{\mathbf{C}}\mathbf{\Delta}_{\mathbf{C}}^{-1}\mathbf{\Pi}^{*}$
$\mathbf{G} \approx \hat{\mathbf{G}}\mathbf{\Delta}_{\mathbf{G}}^{-1}\mathbf{\Pi}^{*}$

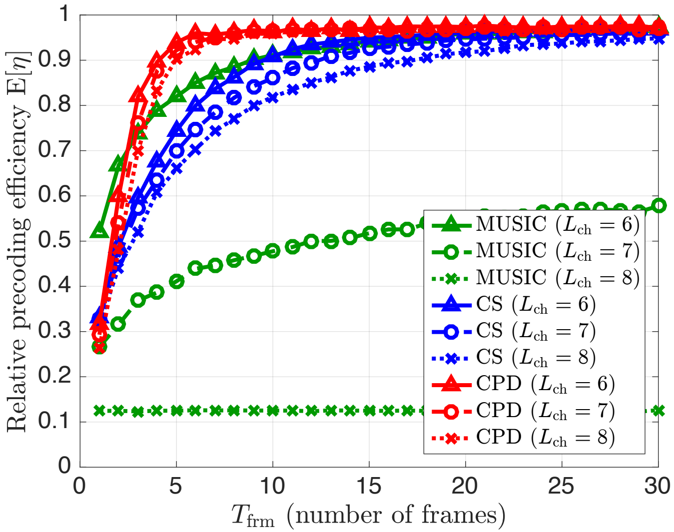
$\mathbf{\Pi}^{-1} = \mathbf{\Pi}^{*}$

$\mathbf{\Delta}_{\mathbf{B}}\mathbf{\Delta}_{\mathbf{C}}\mathbf{\Delta}_{\mathbf{G}} = \mathbf{I}$
$\mathbf{\Delta}_{\mathbf{B}} \approx \hat{\mathbf{\Delta}}_{\mathbf{B}}$

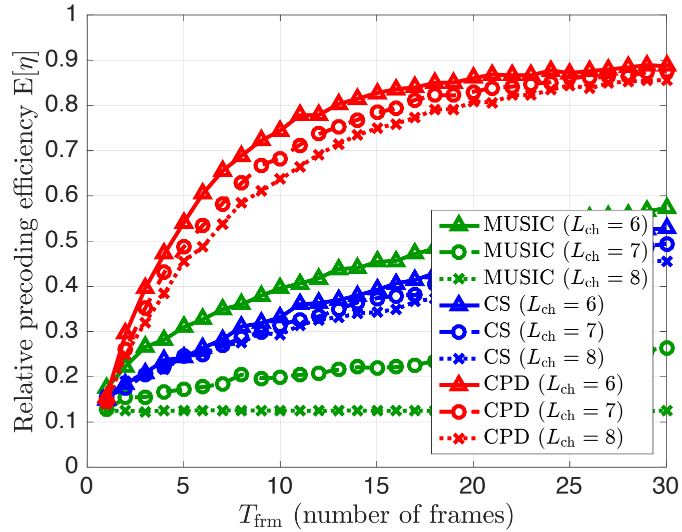
$$\begin{aligned}
\mathbf{R}_{\mathbf{h}} &= \frac{1}{K_{\mathrm{sbcr}}T_{\mathrm{frm}}}\mathbf{A}\left(\mathbf{G}^{*}\mathbf{G} \odot \mathbf{C}^{*}\mathbf{C}\right)\mathbf{A}^{*} \\
&\approx \frac{1}{K_{\mathrm{sbcr}}T_{\mathrm{frm}}}\hat{\mathbf{A}}\mathbf{\Pi}^{*}\left(\mathbf{\Pi}(\mathbf{\Delta}_{\mathbf{G}}^{*})^{-1}\hat{\mathbf{G}}^{*}\hat{\mathbf{G}}\mathbf{\Delta}_{\mathbf{G}}^{-1}\mathbf{\Pi}^{*} \odot \mathbf{\Pi}(\mathbf{\Delta}_{\mathbf{C}}^{*})^{-1}\hat{\mathbf{C}}^{*}\hat{\mathbf{C}}\mathbf{\Delta}_{\mathbf{C}}^{-1}\mathbf{\Pi}^{*}\right)\mathbf{\Pi}\hat{\mathbf{A}}^{*} \\
&= \frac{1}{K_{\mathrm{sbcr}}T_{\mathrm{frm}}}\hat{\mathbf{A}}(\mathbf{\Delta}_{\mathbf{G}}^{*})^{-1}(\mathbf{\Delta}_{\mathbf{C}}^{*})^{-1}\left(\hat{\mathbf{G}}^{*}\hat{\mathbf{G}} \odot \hat{\mathbf{C}}^{*}\hat{\mathbf{C}}\right)\mathbf{\Delta}_{\mathbf{G}}^{-1}\mathbf{\Delta}_{\mathbf{C}}^{-1}\hat{\mathbf{A}}^{*} \\
&\approx \frac{1}{K_{\mathrm{sbcr}}T_{\mathrm{frm}}}\hat{\mathbf{A}}\hat{\mathbf{\Delta}}_{\mathbf{B}}^{*}\left(\hat{\mathbf{G}}^{*}\hat{\mathbf{G}} \odot \hat{\mathbf{C}}^{*}\hat{\mathbf{C}}\right)\hat{\mathbf{\Delta}}_{\mathbf{B}}\hat{\mathbf{A}}^{*}
\end{aligned}$$

Simulation results

SNR 0 dB



SNR -10 dB



Simulation environments

- N_{ANT} (# of antennas at BS) = 64
- M_{RF} (# of RF chains at BS) = 8
- L_{ch} (# of channel paths) = 6, 7, or 8
- K_{sbcr} (# of subcarriers) = 128
- N_{CP} (CP size) = 32
- N_{grid} (# of CS grids) = 256
- Single antenna at MS
- Filter: sinc function (in time)
- SNR = 0 dB or -10 dB
- Analog combining matrix : Random
  . Element-wise amplitude = 1 ($|[\mathbf{W}]_{i,j}|=1$)
  . Element-wise phase : uniform [0~2pi]

Performance metric

sum of eigenvalues (estimated)

$$\frac{\mathrm{Tr}\left(\mathbf{U}^*_{\hat{\mathbf{R}}}\mathbf{R}\mathbf{U}_{\hat{\mathbf{R}}}\right)}{\mathrm{Tr}\left(\mathbf{U}^*_{\mathbf{R}}\mathbf{R}\mathbf{U}_{\mathbf{R}}\right)} \left(= \frac{\sum_{i=1}^{L}\hat{\lambda}_i}{\sum_{i=1}^{L}\lambda_i}\right)$$

sum of eigenvalues (ideal)

MUSIC-based method: Performance degrades rapidly as L_{ch} approaches M_{RF}

CS-based method: Performance is not good at low SNR

Conclusions

Proposed spatial channel covariane estimation for hybrid architectures

Assumption: ***Spatially sparse freqeuncy-selective*** channels

Key ideas: ***Low-rank tensor*** | ***Uniqueness of CPD***

Performance: Good in particualr when

1) ***the number of RF chains decreases*** apporaching the number of channel paths
2) ***SNR is low***

Thank you !